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Imagination and Mathematics in Proclus

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In this article, I provide a comprehensive account of the role of the imagination in Proclus' ontology and philosophy of mathematics. Recently a number of significant studies consider the notion of imagination in ancient philosophy from Aristotle to the Neoplatonic tradition, as well as geometrical objects as intermediate between intelligible forms and sensible things.[1] Dillon 2003, 22-24 has stressed a connection between cognitive modes of the soul, which is a mediator between intelligible and physical realms, and geometrical entities of different dimensions. An important contribution to the consideration of Proclus' account of mathematics in his commentary on Euclid is found in O'Meara 1989, 156-176. In his recent French translation of Euclid (Euclid 1990-2001, 93-113), Bernard Vitrac has provided a helpful commentary pertaining to the investigation of the ontological status of geometrical objects. Mueller 1978, 334-348 has situated Proclus' analysis of mathematics within the Neoplatonic tradition, showing its closeness to, and dependence on, Iamblichus' *De communi mathematics scientia*. A further instructive discussion of a variety of aspects of imagination in Proclus, as well as in ancient, and especially late ancient, philosophy, can be found in the works of Shichalin (Proclus 1994, 18-31), MacIsaac 2001, Watson 1988, and Blumenthal 1978.

Yet there is not much written specifically on the role that imagination plays in the constitution and cognition of mathematical objects in Neoplatonic philosophy and mathematics. I attempt to situate imagination within the structure of cognitive faculties in Proclus, and then consider the imagination's function in the production of geometrical objects and in the attempt to think and constitute the infinite.

In Proclus' argument, imagination, which he boldly identifies with Aristotle's 'passive intellect', provides a *sui generis* materiality for geometrical objects, a kind of 'screen' or 'mirror' onto which discursive thinking (διάνοια) projects mathematical forms, or λόγοι, and makes them 'visualizable' to an inner gaze of the mind. It is in and with the aid of imagination that discursive reason constructs geometrical things by motion. Geometrical objects, which are intermediate between the forms of discursive reasoning and sensible bodies, are taken as fourfold in such a way that every mode of their existence corresponds to a cognitive faculty. Thus, circle is differently present as an intelligible form (the 'notion' of

[1] I follow Morrow's translation (Proclus 1992) except where changes are indicated; all references in parentheses within the body of the text are to Friedlein's edition (Proclus [1873] 1992).

circle in the intellect, or νοῦς), as its appropriation by discursive thinking (διάνοια, the 'definition' of circle), as a geometrical object properly (the 'perfect' circle accessible to imagination (φαντασία) that can be constructed and divided within imagination), and as a physical representation (the image of a circle, which is never properly circular and accessible to sense perception).

Imagination, therefore, is not just one particular notion that is worth studying. But, given the importance of Proclus' commentary on Euclid for our understanding of late Platonic ('Neoplatonic') philosophy in general and of ancient mathematics in particular, Proclus' account of imagination turns out to be a key concept for understanding Platonic-Pythagorean mathematics, and thus allows us to get a better grasp of both the ontological aspect of mathematical being within the general constitution of being, and the epistemological aspect of its cognition within the structure of cognitive faculties.

I. The Intermediate Position of Mathematical Objects

The opening sentence of Proclus' 'Prologue' to his commentary on the first book of Euclid's *Elements* runs: 'Mathematical being necessarily belongs neither among the first nor among the last and least simple of the kinds of being, but occupies the middle ground between the realities that are partless, simple, incomposite, and indivisible—and entities endowed with parts, and characterized by every variety of composition and division' (*In Eucl.* 3.1-7).[1] Mathematical objects are intermediate, 'between' partless simple entities and those that on the contrary have parts and are complex in their makeup. The former, ἀμέριστα, are identical with the plurality of intelligible beings or forms; the latter, μεριστά, are identical with physical things that are subject to change. Mathematical things are as if 'in the vestibule (ἐν προθύροις) of the first forms', and thus ontologically follow εἴδη. Mathematical objects, therefore, are more complex than ideas, but more precise than physical things.[2]

That mathematical beings are intermediate can be seen in that, unlike physical things, they are eternal and are always identical in their properties, yet, unlike intelligible objects, they are presented as partial and divisible through discursive, step-by-step reasoning (*In Eucl.* 3.7-14). In other words, insofar as they are intermediate, mathematical objects share certain features in common with both intelligible forms and physical things, while at the same time are also distinct from each one of them. Therefore, the uniqueness of mathematical objects consists in their being both separate from yet necessarily affiliated with the intelligible and physical, with being and becoming.

That the being of mathematical objects should be taken as μεταξύ, or intermediate, between the intelligible and the physical, is a fundamental ontological and systematic insight of the Neoplatonic tradition with respect to mathematics. Pro-

[2] *In Eucl.* 5.2-3; cf. 55.2. The 'vestibule' allegory comes from Plato's *Philebus* 64c, where Socrates speaks about standing 'in the vestibule of the dwelling of the good'. Proclus also speaks about mathematical objects as intermediate in his *In Timaeuum* ii 23,12-25.

clus ascribes this position directly to Plato, although Plato really suggests that mathematics helps in contemplating being per se (*Rep.* 525a-b; cf. Aristotle, *Meta.* 1090b35-36). There is, however, important evidence in Aristotle's *Metaphysics* i that Plato recognizes the existence of mathematical entities (τὰ μαθηματικά) as being between (μεταξύ) the forms (τὰ εἴδη) and sensible things (τὰ αἰσθητά). According to Aristotle's testimony, mathematical entities differ from sensible things in being eternal and immovable, and from intelligible forms in that every mathematical object can exist in a plurality of similar objects (e.g., circles), whereas each form is one and unique (there is only one form of a circle, *Meta.* 987b14-18; cf. 995b15-18, 997b1-3, and 1028b19-21). Ptolemy in the *Almagest* already speaks about the middle position of mathematical beings (Ptolemy 1898, 6.3-11). Later, Iamblichus in the *De communi mathematica scientia* develops a theory of the intermediateness of geometrical objects that was probably the source for Proclus' discussion of the μεταξύ (Iamblichus 1975, 9.4-12.7; 55.1-8; cf. Philoponus 1867, 2.56: τὰ μαθήματα μέσα ἐστί).

Ontology and epistemology are always intimately connected in (Neo-)Platonism since every kind of being is known in a particular way and through a particular cognitive faculty. Because of this, mathematical beings are intermediate both *ontologically* and *epistemologically*. Ontologically, as was said above, they exemplify properties of both intelligible and physical things. Thus, a whole ontological hierarchy is established among intelligible, mathematical, and physical beings, within which every particular entity finds its place.

At the same time, an ontological sequence or hierarchy directly translates into an epistemological one: every object defines and corresponds to both a cognitive faculty and a corresponding discipline that involves its own set of criteria for precise knowledge about its objects. In such a sequence, knowledge of the 'lower', more complex objects potentially paves the way to an understanding of the 'higher', simpler ones. Thus, an imprecise opinion of physical things makes it possible to present an image that may assist in understanding mathematical things, and a mathematical object or drawing might help in contemplating the intelligibles in their simplicity and entirety. As Plato argues in *Republic* 522c-524e, political knowledge about mathematical entities might open a way for understanding the forms and being per se.

Moreover, different classes of objects define and correspond to different cognitive faculties that are also hierarchically ordered. In particular, the intelligible forms (εἴδη) are known by the non-discursive intellect, or νοῦς, which considers them as its simple, incomposite, and indivisible objects through and by an equally simple act of thinking, which is similar to that of vision. Physical, sensibly perceptible things (αἰσθητά) are conceived of in their incompleteness, transience, divisibility, and partiality of opinion (δόξα), which may be correct but is still always only an unjustified opinion, never strictly grounded, and prone to doubt (Plato, *Meno* 97b-98b, *Phil.* 64a, and Proclus, *In Remp.* i 235.13-21). Intermediate mathematical ideas (λόγοι) are subject to the discursive thinking of διάνοια, which is always partial in the thinking and representation of its object

and moves consecutively, step by step, from the premises of a certain and correct logical order to its conclusion (in *Sophist* 263e Plato famously calls thinking or διάνοια a silent conversation (διάλογος) of the soul with itself; cf. Proclus 1899-1901, i 262.25-29). Νοῦς thus knows its objects/ideas directly and with certainty; διάνοια has to justify its knowledge of intermediate mathematical things by using special procedures and deduction; and δόξα may be right, but even then it is never sure about the correctness of its representations of bodies and their properties.

It is important to note that Plato does not draw a systematic distinction between these cognitive faculties even though he uses the term διάνοια, which refers to thinking in general (*Soph.* 263d-e; *Theaet.* 189e). In the *Republic*, however, Plato distinguishes between νόησις and διάνοια as higher and lower cognitive faculties within the soul, at the same time stressing the affinity of διάνοια with geometry (511d-e; cf. 524b). At times, speaking of reason, Plato also uses the term φρόνησις with νοῦς without making a clear distinction between the two (e.g., *Phil.* 63c). In the simile of the divided line in *Republic* 511b-e, Plato distinguishes between *four* modes of cognition: νόησις, διάνοια, πίστις, and εἰκασία, where only the two first members correspond directly to the Neoplatonic triple hierarchy of beings and their cognitive faculties (see Proclus, *In Remp.* i 283.26-284.2). Thus, Plato provides some hints at the tripartite distinction between intellection, discursive thinking, and sense perception, or νόησις, διάνοια, and αἴσθησις, but never explicitly formulates it. This distinction becomes fundamental and commonplace in Neoplatonism, particularly in Syrianus, who in his commentary on the *Metaphysics* explicitly speaks about three orders of beings: intelligible (noetic, νοητή), discursive (dianoetic, διανοητή), and sensible (aesthetic, αἰσθητή, *In Met.* 879a36-b3).

II. Mathematical Objects: Numbers and Geometrical Figures

When Proclus speaks of mathematical objects, he usually calls them λόγοι, τὰ τῆς μαθηματικῆς εἴδη, or simply τὰ μαθηματικά (*In Eucl.* 4.6-7; 5.11-12; 12.3-4, 14; 16.22 *et al.*). As I have argued already, by situating mathematical objects 'between' intelligibles and physical things, he stresses both the affinity of mathematicals with the two and their distinct identities, which makes mathematical objects unique in their properties. A mathematical object, then, may be called, and in fact is called by Proclus, τὴν μαθηματικὴν οὐσίαν, the mathematical being or substance: such are the very first words of his commentary on Euclid (*In Eucl.* 3.1). As such, a mathematical object is not only taken in its Platonic sense as intermediate between being and becoming (τὰ ὄντα καὶ τὰ γινόμενα, *In Eucl.* 62.25-26), but also as a substance (οὐσία) in the Aristotelian sense, which mediates between opposites that are contrary to one another: i.e., partlessness and constructibility, simplicity and division.

When Proclus calls mathematical things λόγοι, he thereby stresses, first, that they have an inner structure which may become explicit in and as a discursively developed argument, a λόγος; and second, that mathematical objects are con-

ceived in and by discursive reason, διάνοια, whose subjects are precisely λόγοι, with which it is full and of which there is fullness, or πλήρωμα (*In Eucl.* 55.18). For this reason Proclus also calls mathematical things τὰ διανοητά, objects of discursive reason (*In Eucl.* 4.19, 10.23, 62.24 *et passim*). The precision of mathematical forms as objects of and in discursive thinking and argumentative reasoning, i.e., as λόγοι, cannot be explained otherwise than assuming them to be the products (ἔκγονα) of the soul itself, which it derives from reason, or νοῦς, and not by abstraction from physical things (*In Eucl.* 13.22). Otherwise, one would not be able to explain the remarkable precision (ἀκρίβεια) of mathematical objects, for physical things that are subject to (possibly correct) opinion, or δόξα, are in a state of constant becoming and thus themselves do not have any precision that they might communicate or bestow on anything that is derived from them (*In Eucl.* 12.13). Thus, discursive reason (διάνοια) becomes the ultimate 'criterion' (κριτήριον) for mathematics as the capacity to judge about the properties and constitution of mathematical beings (*In Eucl.* 10.16-17, 11.15).

Proclus is well aware, however, that mathematical objects themselves are *double* in kind, represented by numbers and geometrical objects (*In Eucl.* 6.7-7.12; see O'Meara 1989, 166-169). Hence, the structure of the tripartite ontological and epistemological division—three hierarchical layers of being and three distinctive cognitive faculties—has to be rethought and adjusted, which Proclus does in the second part of the 'Prologue'.

The distinction between arithmetic and geometry had already been unambiguously established by Plato. Thus, in the *Republic* Plato differentiates between arithmetic and geometry by reference to their distinctive objects, numbers, and figures, rather than by making a distinction between cognitive faculties (522c, 526d). However, in Plato there is no systematic justification for the distinction between arithmetic and geometry from the perspective of their deduction from some common principles, but rather by reference to the role that mathematical sciences play in education and cognition. Later thinkers and mathematicians, particularly Euclid, considered mathematical investigations to be geometrical *par excellence*, whereas arithmetic rather became a part of a symbolic (Pythagorean) ontology (e.g., in Nicomachus), where numbers stand for being, or a whole plurality of beings in their distinction yet mutual connection. Aristotle, on the contrary, does not make a clear-cut distinction between arithmetic and geometry within mathematics: when he opposes mathematics to theology (first philosophy) and physics, he suggests that mathematics studies those entities that are immovable (ἀκίνητα), not independent (existing separately), and yet related to matter (*Meta.* 1026a6-9).

But what accounts for the distinction between numbers and geometrical figures? Proclus deduces them neither from another kind of object, nor one from the other. Rather, he finds at least two reasons for explaining the difference between numbers and figures. (1) First of all, all beings, including intelligible forms and mathematical things, are governed by the same two principles, the limit and the unlimited (τὸ πέρας καὶ τὸ ἄπειρον, *In Eucl.* 5.18; cf. 131.21-134.7 and

Iamblichus 1975, 12.18-14.17 [τὸ πεπερασμένον καὶ ἄπειρον]). However, the working of the limit and the unlimited is different in different ontological layers: the stronger the presence of the limit, the 'closer' an entity is ontologically to these two principles. The limit provides the properties of a thing with stability and identity. The unlimited is responsible for otherness both in and as the infinite, ἄπειρον, which is already present among intelligible forms both in and as their multiplicity: in number, through the fact that number can always be increased by adding another unit, and in geometrical figures due to their infinite divisibility. The difference between numbers and geometrical figures had already been noted by Aristotle (*Phys.* 207b1-21), to whom Proclus implicitly refers (*In Eucl.* 6.7-19): numbers can infinitely (i.e., indefinitely) increase but cannot infinitely decrease, because their 'lower' limit is the unit, or μονάς. On the contrary, geometrical figures can be infinitely divided, because there is no smallest part in a geometrical figure as a continuous quantity. Yet a geometrical figure cannot be increased because otherwise it would turn into another object of the same kind but quantitatively different. Therefore, a particular figure is its own limit from 'above'. Thus, arithmetical numbers and geometrical figures behave differently with respect to infinite increase and decrease. This means that the first principles of mathematical being are different for different beings. Moreover, these principles are deducible, or come from (τὸ ἀπ' ἄλλων ἄρχων) the first two principles, the limit and the unlimited, that are present in and at work in all things (*In Eucl.* 3.12).

(2) Furthermore, all numbers are commensurable: their common measure is the unit, whereas not all geometrical figures of the same kind are commensurable but may display mutual inexpressibility and irrationality (ἄρρητον, ἄλογον, *In Eucl.* 6.21; cf. Euclid, x, def. 3-4 and Heath's commentary in Euclid 1956, 11-13). This means that geometrical entities are present in and connected with matter or materiality, and that the measure of precision is different in numbers and in figures. In other words, there is a certain inalienable otherness in geometrical figures that cannot be overcome and subdued by the same of the limit and that makes geometrical things similar to physical ones. The number 'one' in arithmetic and the point in geometry pose a problem in that each of them, being indivisible, stands apart from other numbers and geometrical entities, respectively, and may be considered as both a 'generative unit', on the one hand, and as a limit with respect to objects of their kind, on the other (cf. *In Eucl.* 85.1-96.15).

Because of this, the very principles of figures and numbers (σχημάτων καὶ ἀριθμῶν) are generically different (*In Eucl.* 33.5-9; cf. 59.15-16). It is precisely this distinction that underlies Proclus' deduction of the Pythagorean quadrivium (*In Eucl.* 35.19-36.3), where the difference between τὸ ποσόν and τὸ πηλίκον ('quantity' and 'magnitude') matches that between numbers and geometrical figures. However, the ποσόν/πηλίκον distinction is broader because ποσόν embraces the objects of arithmetic (quantity considered by itself) and music (quantities considered in relation to another), while πηλίκον encompasses those of geometry (unmoving magnitude) and astronomy (magnitude in motion, cf.

Nicomachus 1866, 4.13-7.2, Iamblichus 1975, 29.22-31.4, and Iamblichus 1975², 7.2-9.1; for a more detailed discussion, see Vitrac 2005).

Thus, both arguments that account for the separation and distinctness of arithmetic and geometry as disciplines refer to the difference in the presence and appearance of otherness as *materiality* and *infinity* in numbers and figures. In particular, unlike numbers, geometrical figures are infinitely divisible and associated with a kind of matter, which makes them extended and perceptible as visualizable and extended.

III. Phantasia as a Fourth Cognitive Faculty Related to Geometrical Objects

What does it mean, however, that geometrical figures may be visualized or 'seen'? Physical things are seen as bodies, i.e., are perceived sensibly. And yet, as Proclus has argued, they can never be precise, whereas geometrical figures must be perfect, i.e., adequately represent their corresponding properties. Thus, a straight physical line is never straight, and a bodily circle is never round, whereas a geometrical straight line cannot be anything else but straight, and a circle nothing but perfectly round, which follows from their definitions. Therefore, sense perception, or αἴσθησις, cannot be the faculty responsible for the adequate representation of geometrical figures. Discursive reason, however, conceives geometrical objects in their properties as λόγοι, which are not extended. This means that there has to be a distinct cognitive faculty capable of representing geometrical objects as figures, i.e., as extended *and* perfect. Proclus calls this faculty φαντασία, or imagination, which is not mentioned in his initial discussion of the cognitive faculties but is introduced toward the end of the first part of the 'Prologue' and discussed extensively in the second part.[3] Here, φαντασία cannot be identified with Plato's εἰκασία, and sense perception, or αἴσθησις, cannot be identified with πίστις, because in Plato's order of νόησις-διάνοια-πίστις-εἰκασία in the *Republic*, πίστις comes 'before' εἰκασία in the order of cognition that is relevant for understanding the forms and geometrical objects (511d-e; cf. *Tim.* 29b-c). Yet, in the structure of the cognitive faculties, as Proclus suggests in his commentary on Euclid, the order is precisely the reverse.

In this way, φαντασία becomes the *fourth* cognitive faculty in the order of νοῦς, διάνοια, φαντασία, and αἴσθησις, i.e., intellect, discursive reason, imagination, and sense perception (cf. Proclus, *In Remp.* ii 277.18-19). By placing imagination between discursive reason and sense perception, Proclus directly follows Aristotle's famous discussion of imagination in *De anima* iii 3 (φαντασία γὰρ ἕτερον καὶ αἰσθήσεως καὶ διανοίας, 427b14-15). That cognitive faculties form a hierarchy was a common doctrine in Neoplatonism since Plotinus. In particular, the hierarchy of discursive reason, imagination, and sense perception (διάνοια, φαντασία, and αἴσθησις) was already established by Themistius in

[3] *In Eucl.* 45.5-15, esp. 51.9-56.22. See Charles 1971 and MacIsaac 2001, 125-135. However, *contra* MacIsaac, φαντασία is *not* a type or mode of διάνοια. Together with discursive reason, imagination is *in* the soul, but nowhere does Proclus argue that imagination is contained in, or forms a part of, discursive reason.

his commentary on Aristotle's *De anima*, and was later supported by Simplicius (see Blumenthal 1978, 253-256).

In Proclus, imagination or φαντασία becomes responsible for representing and constructing geometrical forms or figures (τῶν γεωμετρικῶν εἰδῶν) and for furthermore submitting them to the consideration and study of discursive reason, or διάνοια (*In Eucl.* 57.5; 285.21, *et al.*). Because of this, geometry is preceded by arithmetic, which is paralleled by imagination, which in turn is preceded by discursive reason. Imagination, then, is intermediate between sense perception and discursive reason: with the former, imagination shares the capacity to represent geometrical figures as extended; with the latter, it shares the capacity to represent its object as unchangeable according to its properties.

Nevertheless, geometrical figures are represented in and by each of the four faculties, albeit differently by each. First, then, a geometrical figure such as a circle may be said to be both one and fourfold at the same time. Thus, a circle exists as a sensibly perceptible physical image (ἐν τοῖς αἰσθητοῖς), which is imperfect, imprecise, and exemplifies straightness. Moreover, second, a circle also exists as a perfectly circular figure in imagination (ἐν φαντασίᾳ), where it is infinitely divisible (i.e., can be divided continuously into parts that are themselves divisible), formed, and extended (μεριστὸς ἐσχηματισμένος διάστατος). This is a geometrical figure in the proper sense. Third, in discursive reason a circle exists as a λόγος or εἶδος τῆς μαθηματικῆς, where it cannot be visualized, is one, simple, non-extended, and devoid of magnitude. As a λόγος it is present in thinking without matter, and hence its very magnitude is without magnitude, and its figure is without figure or shape (τὸ μέγεθος ἀμέγεθες, and τὸ σχῆμα ἀσχημάτιστον) (*In Eucl.* 54.5-13). One may say that a circle as λόγος exists as a definition. An example of this would be: 'a circle is a plane figure constituted by all the points equidistant from a given point', or 'a circle is a plane figure constructed by the motion of a point at a given distance (radius) around a given point (center)'. As such, a circle is devoid of matter, shape, and extension, even though it has distinct and discrete parts in its definition. Obviously, such a λόγος is neither extended nor circular, but still includes all of the properties of a circle that may be shown or demonstrated in various propositions. And finally, fourth, a circle exists as a single, unique, and unified εἶδος, which is not a mathematical εἶδος, but rather a notion. Unlike the image of imagination, this notion is not extended and cannot be visualized; but unlike the λόγος, it is partless even though it wraps all of the circle's properties into a simple act of comprehension by the intellect, or νοῦς. Νοῦς contains everything about a circle 'in concentration' (συνῃρημένως), whereas soul contains it in an ordered, step-by-step thinking and obtains it discursively (διῃρημένως, *In Eucl.* 16.10-16). At each ontological and cognitive level—sensible, imaginary, dianoetic (discursive), and noetic (intelligible)—a circle remains the same circle, and yet is differently exemplified. One may say that each exemplification of a circle offers itself as a clarification and *sui generis* illustration of the next one, up to the real circle whose form is grasped and thought by νοῦς. Therefore, the form of a circle is not itself circular;

instead, the extended and circular geometrical circle resides in imagination.

IV. Phantasia and Materiality

As λόγος, a circle exists (1) as a definition and (2) in a discursive, structured, logically ordered, and correct proposition—as an argument that discloses its intrinsic properties. To reiterate, the λόγος of a circle is not itself circular, i.e., is not itself a geometrical figure. The geometrical, extended, and divisible object that exemplifies its properties without aberration (i.e., is properly circular in the case of a circle) is present in imagination as a figure (σχῆμα) which is a 'picture' of its λόγος. This perfect yet extended circle that is conceived or 'seen' in and by imagination facilitates discursive logical reasoning about the circle and its properties.

It is important to note that a circle in both sense perception and imagination is *plural* and thus presupposes infinity because there can be any number of physical and imaginary circles, whereas there is only one circle in discursive reason and intellect, i.e., only one λόγος (bound by all of its properties that are already there but need to be discovered and disclosed by discursive logical reasoning), and one εἶδος (notion) of a circle. The difference between the representation of a circle as dianoetic, imaginary, or sensual is that a circle as λόγος is one, whereas as imaginable it is not one but rather both one and many (οὐχ εἷς μόνον, ἀλλ' εἷς καὶ πολύς, *In Eucl.* 54.9-10), and as sensible it is multiple only, both in number and according to the ways in which the initial unity and simplicity of a circle is distorted.

As multiple and multiplied, on the one hand, and as extended and divisible in such extension (e.g., in segments) on the other, a circle should be conceived as existing in a certain *matter*. Here one must distinguish between physically embodied, materialized figures, which are imprecise and *remind* us of the properties they bear rather than adequately represent them to discursive reason, and figures in the soul, which are also conceived in and by imagination, the latter of which thereby provides and serves as a special 'place', or τόπος—a particular matter that is capable of accommodating extended figures (cf. *In Eucl.* 15.5-6; 51.9-20).

Therefore, as Proclus argues, one must assume the existence of intelligible or geometrical matter, because there are no geometrical figures either in the physical world (where they are all imprecise) or in discursive thinking as pure λόγοι (λόγοι καθαροί, *In Eucl.* 49.25), which do not have (infinitely divisible) parts (*In Eucl.* 49.12-26). It is thus only in mathematical or geometrical matter (ὕλη τῆς γεωμετρίας) that a geometrical figure has parts, is infinitely divisible, extended, and is also precisely what it is, holding all of its properties open to the 'gaze' of discursive reason without distortion. Only in geometrical matter (κατὰ τὴν γεωμετρικὴν ὕλην) is the point without parts and the circle really circular (*In Eucl.* 93.18-19).

When Proclus distinguishes two matters, physical and geometrical, which 'locate' two classes of things, sensible and geometrical (τὰ μὲν αἰσθητὰ τὰ δὲ ἐν

φαντασίᾳ τὴν ὑπόστασιν ἔχοντα), he follows Aristotle, whom he interprets within the context of a Platonic understanding of mathematics (*In Eucl.* 51.14-15). Aristotle mentions intelligible matter as present in mathematical things in opposition to matter in sensible things (ὕλη νοητή and ὕλη αἰσθητή).[4] Plato, on the contrary, does not make such a distinction anywhere in his dialogues, nor does he establish any link between materiality and imagination. At the same time, the notion of φαντασία appears in Plato and is distinguished from διάνοια and δόξα (*Soph.* 263d6), where, however, different cognitive faculties neither refer to nor define the various sciences as they do in Proclus.

To sum up, multiplicity is present in and through a circle's extension, divisibility, and materiality. A specifically geometrical matter, then, accounts for both the extension of geometrical objects and their possible incommensurability with one another (inexpressibility and irrationality).[5]

V. Φαντασία and Motion

The association of an object with matter means that the object remains forever incomplete, and that it never 'coincides' with itself as it is defined by its definition and notion, or λόγος and εἶδος. Furthermore, materiality implies an association with motion whereby a thing moves toward its fulfillment and completion, which, however, it might never obtain. In the case of geometrical objects, an association with matter also means a connection with motion. But, unlike the motion of physical things, geometrical motion is a *sui generis* becoming rendered as being, or incompleteness made complete.

Geometrical motion is productive, so it is neither locomotion nor motion with respect to quality. Because mathematical objects, as was argued above, are subject to the consideration of discursive reason and imagination, motion is therefore present in both. In discursive reason, or διάνοια, it is the motion of thinking and reasoning from premises to conclusion, whereby one λόγος of a mathematical thing is completed by being unwrapped through a number of steps and thus clarified in and by means of its proof. This is a 'life-giving' motion (ζωτική), which makes a mathematical object come alive in the life of the mind (*In Eucl.* 18.20-28). In this respect, imagination is a 'shape-producing thinking' (cf. Proclus, *In Remp.* i 235.18-19: ἡ...φαντασία νόησις οὖσα μορφωτική) that enables discursive thinking to come to a stop and arrive at the 'recollection' (ἀνάμνησις) of what it already has, i.e., the knowledge of the εἶδος of a (mathematical) thing (*In Eucl.* 46.3-47.5; cf. *Meno* 81b-86c, *Phdo.* 72e-76c, *Phaedr.* 249b-c, *Phil.* 34b-c).

Furthermore, motion is present in imagination. Thus, a circle as an image of imagination may be taken as a 'whole' that exists all at once in an *act* of imagining. Such an image represents the form (notion) of a circle and yet, unlike the

[4] Aristotle, *Meta.* 1036a9-12. It is worth noting that in the critical apparatus Jaeger mentions that this particular sentence referring to geometrical matter appears to have been added to the text at a later time. See Aristotle 1969 *ad loc.*

[5] For a more detailed discussion, see Nikulin 2002, 183-187.

form of a circle, it is not simple because the figure as an image of imagination is extended and infinitely divisible. At the same time, a circle in the imagination may be taken in the *process* of its construction on the part of the motion of one geometrical object (a point) in imagination that traces the circle, which is thus produced by the continuous (not discrete, as in a number of steps in an argument or proof) 'flow' of the point (cf. *In Eucl.* 185.25-187.3).

Therefore, motion in imagination is motion that may be taken as producing one geometrical object in and by the *motion* of another (point, line) that traces its movement in imagination through geometrical or intelligible matter (ὕλη νοητή, see Nikulin 2002, 245-254 and Harari 2006, 383-387). It is in imagination, then, that all kinds of such kinematic constructions and divisions take place, which facilitates the understanding of the properties of a figure by making them quasi-visual. It is here, in imagination as the locus of geometrical ideas, that the λόγοι are formed and constructed into an imaginable figure that is thoroughly studied, and then is as though returned or brought back to the non-visualizable simplicity of its λόγος in both discursive and non-discursive thought (*In Eucl.* 78.18-79.2). This constructive motion in imagination is a kind of production of a figure, which may engender a whole plurality of imaginable representations (e.g., many concentric circles from the single λόγος of a circle), and yet fully preserves the properties of the geometrical object.[6] It is here in the intelligible matter of imagination that the constructed circle is perfectly circular and the halves of a divided line actually identical in length.

VI. *Phantasia* as νοῦς παθητικός

Imagination is thus connected with shape-producing motion and some kind of materiality or body;[7] it carries and produces images that it can embody, make visible, and allow to be considered by discursive reason. Proclus identifies imagination as such with the 'passive intellect', or νοῦς παθητικός (*In Eucl.* 52.3-12; 56.17-18). It is worth noting that this identification becomes widely accepted after Proclus in the Neoplatonic commentaries on Aristotle, particularly in Asclepius, Simplicius, Stephanus, and Philoponus (see Blumenthal 1978, 255).

Proclus thus uses two key terms to describe Platonic mathematics that are found in Aristotle but are absent from Plato's own writings. Imagination, then, is the geometrical or 'intelligible' matter of and for geometrical figures that coincides with the passive intellect. Proclus refers to the famous passage in Aristotle's *De anima* that has caused so many debates and provoked lengthy commentaries over the centuries. Here Aristotle suggests that one can distinguish various aspects of the intellect, perhaps that in actuality from that in potentiality

[6] εἰς ἐκείνην οὖν [σψ. νοητὴν ὕλην] οἱ λόγοι προϊόντες καὶ μορφοῦντες αὐτὴν εἰκότως δήπου ταῖς γενέσεσιν ἐοικέναι λέγονται. τὴν γὰρ τῆς διανοίας ἡμῶν κίνησιν καὶ τὴν προβολὴν τῶν ἐν αὐτῇ λόγων γένεσιν τῶν ἐν φαντασίᾳ σχημάτων εἶναί φαμεν καὶ τῶν περὶ αὐτὰ παθημάτων (*In Eucl.* 78.20-25).

[7] καὶ γὰρ ἡ φαντασία διά τε τὴν μορφωτικὴν κίνησιν καὶ τὸ μετὰ σώματος καὶ ἐν σώματι τὴν ὑπόστασιν ἔχειν (*In Eucl.* 51.20-22).

and from νοῦς παθητικός (430a10-25, esp. a24-25).

At first glance, the very term, 'passive intellect', seems to be an oxymoron for Proclus (*In Eucl.* 52.4-6): intellect cannot be passive because it is pure activity, an *act* (not a discursive process) of thinking that thinks itself. However, qua νοῦς παθητικός, imagination embraces its opposites without violating the principle of non-contradiction: imagination is both active and passive, but in different respects. Indeed, on the one hand, as νοῦς, imagination is active in that, first, it does not distort that which it represents. Second, imagination produces its images (οἰστική, *In Eucl.* 52.2); and third, it is 'woken up by itself' (ἀνεγείρεται...ἀφ' ἑαυτῆς), i.e., capable of self-motivation and *sui generis* autonomy (which is of central importance not so much to mathematics but to art and fiction, *In Eucl.* 52.22). On the other hand, imagination is passive, παθητικός, because it is associated with materiality or matter, which in Platonism is considered as nothing by itself and as not having anything of its own—no form of and within itself, no concealed potential that matter might reveal by itself. Matter always needs an active exterior component in order to be formed: matter is always formed, and never forms itself. Grammatically, only the passive voice does justice in describing the state of matter, which is nothing by itself or on its own.

Thus, Proclus chooses to use the term 'passive intellect' because, on the one hand, he wants to stress the middle position of imagination between becoming and being, between sense perception and discursive thinking (*In Eucl.* 52.8-12). On the other hand, cognitive faculties are defined by their objects, and imagination is determined in mathematics by geometrical figures, which have, as was said above, the features of both 'passive' physical bodies (as extended and divisible), and of thinkable λόγοι (through identity and unchangeability), yet are neither of the two. The very object of imagination, therefore, is the object of and for νοῦς παθητικός.

VII. *Phantasia* and Projection: Imagination as Screen and Mirror

What, however, is the importance of imagination for mathematics? The task of a mathematician is to study mathematical forms, or λόγοι, in their various properties and mutual relations. As was said earlier, discursive reason, or διάνοια, thinks of its object, or λόγος, as being always and inevitably partial, in which sense it is distinct from the intellect, or νοῦς, which is capable of thinking its object, εἶδος, in its entirety. This means that discursive thinking is a weak form of thinking, because, although it already has all of the λόγοι, it is incapable of thinking and 'seeing' them in their entirety all at once and 'enfolded' (ἀσθενοῦσα...συνεπτυγμένως ἰδεῖν, *In Eucl.* 54.27-55.3). Therefore, discursive reason has to 'unwrap' or 'unfold' (ἀνελίττειν) that which is enfolded in each λόγος in order to be able to investigate it in an orderly manner.

Discursive reason has two ways of overcoming its feebleness: dialectic and construction in imagination, which ultimately converge, since mathematics must

lead to dialectic and eventually to the understanding of being.[8] Every λόγος is conceived in and by a logically structured discursive argument that presupposes a number of closely related dialectical procedures (Proclus distinguishes between analysis, synthesis, definition, and demonstration). Dialectic 'unwraps' a λόγος by showing it in its definition and demonstrating its properties (cf. *In Eucl.* 42.9-43.21). Yet in dialectical disclosure λόγος is never given in its entirety but rather only partially at each step and moment of thinking, similar to a piece of music being heard at each moment only in part.

At the same time, διάνοια is capable of representing the λόγος of a mathematical object all at once. The price for this capacity is that the λόγος, then, becomes something alien to and different from itself: it becomes an image of and in imagination (τὸ φανταστόν, *In Eucl.* 51.19) that is an embodiment of an immaterial and invisible λόγος in the geometrical or intelligible matter of imagination. Such an image is a *sui generis* painted picture that is accessible in all of its detail and components at a glance. Everything that is contained in a mathematical form as latent, hidden, 'secret' or concealed (κρυφίως, *In Eucl.* 56.13), becomes visible in and through its image in imagination. Such a picture 'unfolds' a λόγος, for instance as an imaginable circle, which is a φανταστόν of the λόγος of a circle (of its definition as 'all of the points equidistant from a given point'). This image, again, may be realized either as an *act* (of visualizing a circle in imagination), or a *process* (of tracing a circle by the motion of a point in imagination as geometrical matter).

'Every true geometer', explains Proclus,

> should...make it his goal to wake himself up and move from imagination to pure discursive reason as it is by itself, thus rescuing himself from the extension and 'passive intellect' for the sake of the dianoetic activity that will enable him to see all things as non-extended and without parts—the circle, diameter, polygons [inscribed] in the circle, all in all and each separately (*In Eucl.* 55.23-56.4, trans. mod.).

Imagination thus facilitates discursive reason's study of a mathematical object, which, as form or εἶδος, is unmoved, not produced, and indivisible (ἀκίνητον, ἀγένητον, ἀδιαίρετον, *In Eucl.* 56.12), but which has to be 'alienated' from itself, set into (imaginary) motion, brought into an (imaginable) extension, and placed in (geometrical) matter. A 'drawing' or σχῆμα of a mathematical entity is, then, a figure in imagination that helps the geometer to recognize it in a physical drawing and further understand it as λόγος with all of its properties and connections to other geometrical entities (e.g., a circle and an inscribed right triangle, cf. *In Eucl.* 142.9-12).

In Plato, both numbers and figures are considered and studied according to

[8] Proclus explicitly mentions Plotinus, who in his treatise *On Dialectic* says, implicitly referring to *Rep.* 531c-534e, that following mathematical studies one must turn to dialectic in order to become a complete dialectician (Plotinus, *Enn.* i 3[10]3.1-10; Proclus, *In Eucl.* 21.21-24; cf. 32.7-20).

their properties by discursive reason, or διάνοια. As Aristotle mentions in *Metaphysics* i 6, ideal mathematical numbers, and not geometrical figures, remain the primary objects of thinking for Plato. In the *Euthydemus*, however, Plato mentions that the mathematician uses drawings (τὰ διαγράμματα), which assist thought through an imaginary 'seeing' or visualization of the properties and demonstrations for which it is searching (*Euthyd.* 290b-c; cf. *Meno* 73e).

This act or process of picture-like 'visualization' of a non-extended and indivisible mathematical object, or λόγος, as a geometrically extended and divisible figure or φανταστόν may be compared to a novel on film. Reading and understanding a novel is a discursive activity that involves the attentive gathering of the meaning of what is read and thereby undergone, whereas on film the story is represented as a series of pictures and images projected onto a screen. To continue this analogy, in order to understand itself and its contents, the διάνοια must become a painter or projector: it has to project or 'throw forward' (προβάλλειν) the λόγος onto the screen of imagination, watch it there, understand and study it and its properties (which, despite the simplicity of mathematical form as such, may be rather complex), and then bring the λόγος back to itself in its unity, simplicity, non-extension, and indivisibility. Διάνοια, then, is that which projects (τὸ μὲν προβάλλον ἡ διάνοια); that which is projected is the dianoetic discursive form (τὸ δὲ ἀφ' οὗ προβάλλεται τὸ διανοητὸν εἶδος); and that onto which it is projected is the so-called 'passive intellect' (τὸ δὲ ἐν ᾧ τὸ προβαλλόμενον παθητικὸς οὕτος καλούμενος νοῦς, *In Eucl.* 56.15-18; cf. 17.4-6).

Now all of the terms of the geometrical screening are brought together: a geometer must look at the images of the mathematical form that discursive reason unfolds by painting or projecting it onto the geometrical matter or 'passive intellect' of imagination, make all the necessary divisions and constructions in such an image, discover its properties, and then enfold and bring it back into its mathematical form as being already undivided and only thinkable.

Because Proclus has this logical and imaginary sequence of the clarification of λόγος in mind, he also speaks about imagination as a plane *mirror* (οἷον ἐπιπέδῳ κατόπτρῳ) into which discursive reason 'looks' and recognizes itself as that 'to which the λόγοι of the discursive reason send down reflections (ἐμφάσεις) of itself' (*In Eucl.* 121. 4-7, trans. mod.). That imagination is a plane mirror means that it is a kind of 'smooth surface' that, unlike the mirror of bodily matter, does not distort the projections of discursive reason that appear as geometrical figures. Hence, 'the soul, exercising her capacity to know, projects (προβάλλει) onto the imagination, as on a mirror (ὥσπερ εἰς κάτοπτρον), the λόγοι of the figures (τοὺς τῶν σχημάτων λόγους); and the imagination, receiving in images and as reflections those entities that the soul has within itself, by their means affords the soul an opportunity to turn inward (εἰς τὸ εἴσω) from the images and attend to herself. It is as if a man looking at himself in a mirror (ἐν κατόπτρῳ) and marvelling at the power of nature and at his own appearance (μορφήν) should wish to look upon himself directly and possess such a power that would enable him to become at the same time the seer and the seen (ὁρῶν καὶ ὁρατόν)' (*In Eucl.*

141.4-13, trans. mod.).

By making its λόγοι external and reflected in the 'mirror' of imagination, discursive reason returns to itself and to pure dianoetic activity (πρὸς τὴν διανοητικὴν ἐνέργειαν, *In Eucl.* 56.1) by understanding the fullness (πλήρωμα, *In Eucl.* 55.18) of its objects (λόγοι), which originally existed independently yet are connected with each of the others, and are now embodied separately as 'seen' in the medium of imagination, non-physical matter, and extension. In this way, διάνοια turns back toward itself and thus becomes reflective; its reflectivity is not immediate, as is the case with νοῦς, but is rather obtained either through dialectical argument or in an effort to investigate its own constituents as visualized and imaginable representations (*In Eucl.* 55.23-56.22).

At this point, it is important to note that, being capable of a reflective return to itself, discursive thinking moves itself, or is moved by itself toward externality (ἀφ' ἑαυτῆς εἰς τὸ ἔξω κινουμένης) in order eventually to come back to itself (*In Eucl.* 55.12). It is the 'projector', τὸ προβάλλον, that which projects the images of the λόγοι as images in and onto imagination. At the same time, Proclus explicitly says that imagination projects or 'throws forward' that which is known (προβάλλει τὸ γνωστόν), and in this way sets what it is looking at into motion (*In Eucl.* 52.22). This should mean that both discursive reason (διάνοια) and imagination (φαντασία) take part in the exteriorization and making visible of the otherwise interior (to thinking) and invisible (only thinkable) mathematical form or λόγος. The act of production, of 'throwing' what is known into geometrical matter, is thus a *joint activity* of imagination and discursive reason.

Imagination, then, is not altogether passive but also active insofar as it presents the mathematical λόγοι to διάνοια in such a way as to facilitate their consideration by imagination as a 'smooth mirror' that reflects them. By being capable of producing a figure in motion within itself, imagination also makes mathematical objects visualizable and visible to the 'eye of the mind' as extended images. Διάνοια thus deploys and unwraps (*In Eucl.* 55.3) its λόγοι not only in but also with the help of imagination, which means that imagination is not an altogether passive matter of and for geometrical objects, but, being geometrical or intelligible matter, also actively contributes to the 'well-being' of mathematical forms as precise and undistorted geometrical figures. Therefore, one may say that φαντασία exemplifies features of both reproductive and productive imagination: it reproduces a λόγος projected into it by discursive reason, but it also actively contributes to a kinematic production in and as a geometrical figure.

VIII. *Phantasia* and the Infinite

Once a geometrical figure is constructed in imagination as a projection of a mathematical form, it is conceived as definite and therefore limited. As I have argued above, such a construction is achieved by both discursive reason *and* imagination. In fact, Proclus suggests that imagination thinks (νοεῖ) its object, which he calls φάντασμα or φανταστόν, and thinking is precisely an act that brings to bear both form and limit, μορφὴν...καὶ πέρας, 'and in knowing brings

to an end its movement (διέξοδον) through the imagined object; it has gone through and comprehends it' (*In Eucl.* 285.7-10). In other words, the constructive motion of imagination is finite because it produces finite objects in imagination as geometrical figures and as such comprehends them.

In certain geometrical problems, however, one needs to make a construction that presupposes an *infinite* line. Proclus speaks about infinity in its relation to imagination when commenting on proposition XII of book 1, which is a problem that requires the construction of a perpendicular line on a given infinite line from a given point which does not lie on that line (*In Eucl.* 284.17-286.11).

Thus, infinity is indispensable for and in geometry, where it is utilized for construction. That a given line is infinite does not mean that it is already there as infinite, i.e., that it is actually infinite—but only that it can always be extended further if needed, i.e., that it is potentially infinite. The infinite, τὸ ἄπειρον, is therefore that which can always be otherwise (bigger or smaller) than it is. As such, the infinite is not a particular object or thing, which would mean that it would be always definite and limited. The infinite, rather, must be associated with becoming, which in geometry is present in the construction and production of an object for further study. Therefore, the infinite should be linked to *matter*, which is indeterminate.

As indeterminate, matter does not have a proper notion, but rather a vague concept that is referred to as 'infinite'. As Aristotle says, the infinite as matter is unknowable (ἄγνωστον), because matter has no form (εἶδος γὰρ οὐκ ἔχει ἡ ὕλη, *Phys.* 207a25-26). In other words, matter is *formless*. Similarly, in the *Timaeus* Plato argues that matter, as the 'receptacle' of things, is indeterminate (49a-52d, esp. 51a, 52a). In his own account of matter, Proclus follows both Plato and Aristotle: as the 'receptacle' and 'seat' of things (ὑποδόχη, ἕδρα, *In Eucl.* 15.12-13), matter cannot be determinate or definite. Plato, however, does not recognize a distinct matter or 'receptacle' for geometrical things, yet the infinite in geometry is associated not with sensible things that are incapable of adequately representing geometrical properties (straightness, circularity, etc.), but with geometrical figures. Hence, the geometrical infinite for Proclus must be linked to imagination, because, as was argued above, imagination is geometrical matter (ὕλη γεωμετρική), which in turn is the locus of projection and existence for geometrical entities.

But where is the infinite present? As matter, it cannot be said to properly be or exist. It occurs only as a possibility, particularly as the possibility of a figure to be constructed or a straight line to be extended. In *Physics* 207a21-25, Aristotle claims that the infinite is matter for the completion of magnitude (ἔστι γὰρ τὸ ἄπειρον τῆς τοῦ μεγέθους τελειότητος ὕλη) and in its entirety exists only *potentially*, appearing in and through divisibility and the increase of magnitudes. As Aristotle argues (accepted by Proclus), sensible things cannot be infinite because they are limited in their existence, bodily makeup, power, and motion.[9] But intel-

[9] *Phys.* iii 4-8.202b30-208a23, esp. 204a8-206a8; cf. Proclus, *Inst. Phys.* prop. I, 12-13, II, 6-15

ligible things cannot be infinite either, because they too are limited and definite, and, as forms, are the source of definiteness in geometrical figures and sensible magnitudes as their representations. Therefore, the infinite for Proclus can only be present 'between' the sensible and the intelligible. Hence, once again, the infinite has to be associated with imagination as an intermediate faculty that represents entities that are themselves intermediate between sensible bodies and dianoetic λόγοι. As indeterminate, the infinite *is* imagination, which, as geometrical matter, can become any figure and can accommodate the further extension of a line without it ever becoming actually infinite.

The situation, in fact, is even more complex because, on the one hand, the infinite as indefinite potentiality is identical with matter as the indeterminate capacity to become anything that a λόγος or εἶδος makes of it. As such, the infinite is non-being; it is nothing of and by itself. On the other hand, the infinite as indefinite is one of the two principles, the limit and the unlimited (τὸ πέρας καὶ τὸ ἄπειρον), which, as Proclus argues in the very beginning of his commentary on Euclid, are the very principles of everything, including mathematical entities (*In Eucl.* 5.18; cf. *Theol. Plat.* iii 8 [Proclus 1968, 30.15-34.19]). Therefore, the infinite appears both as non-being *and* as one of two intelligible, perfectly definite, and thinkable principles of all being. Providing an explanation of the relation between these two radically different ways in which the infinite is present in and to all things presents a challenge for Proclus: it is a difficult systematic problem that is never addressed in his consideration of mathematics because it belongs to the discussion of first principles, unity, and multiplicity.[10]

But how is the infinite constituted and how is it known? The infinite is not an object; it is not a thing that can be known as a particular object of cognition, which must be limited whereas the infinite is not limited. At this point, Proclus makes an extremely original move: the infinite is known and constituted in its 'what' in a purely negative way. The infinite, then, is that which is not thought but is that about which imagination cannot decide what it is; imagination is *uncertain* as to what it is and thus cannot define it (ἀοριστα ινούσης περὶ τὸ νοούμενον). Hence, the infinite is that about which imagination 'suspends further thinking, and calls infinite everything that it abandons, as immeasurable and incomprehensible by thought (ὅσον ἀκαταμέτρητον ἀφίησι καὶ ἀπερίληπτον νοήσει)' (285.10-13). Whatever imagination 'dismisses as something that cannot be gone through, this is calls infinite (ὃ γὰρ ἀφῆκεν ὡς ἀδιεξίτητον, τοῦτο ἄπειρον λέγει)' (285.18-19).

Consequently, the infinite is constituted purely negatively: it 'is' what an object of thinking cannot be. Infinity is thus a 'missed subject'. The infinite is the unalienable other of a limited (geometrical) object, also constituted in and by imagination. The infinite is a 'construction' by imagination of that which cannot

et passim and see Nikulin 2003.

[10] Proclus mentions henads as the source of the multiplicity of figures (*In Eucl.* 142.5-6), but does this in passing without elaborating the topic much further. Cf. *Theol. Plat.* iii 5-6 (Proclus 1968, 17.11-28.21) and *El. Theol.* props. 113-165 (Proclus 1963, 100.5-144.8).

be constructed. It is the mere possibility of going further, of taking another step in the extension or division of a figure's multiplication.

Such a possibility of always being otherwise is manifested as imagination's 'indivisible power of proceeding incessantly, without end' (*In Eucl.* 285.15-17). The source or 'foundation' (ὑποβάθρα) of the infinite, then, is the very boundlessness of imagination (*In Eucl.* 286.10-11), whereas thinking is definite. It is the 'materiality' of imagination that allows an immaterial mathematical form to become 'alienated' or 'projected' into matter, which acquires features of otherness in association with the unlimited. Metaphorically, 'knowing' the infinite is similar to 'seeing' darkness in dark. As an activity, seeing exists and is capable of seeing anything, but there is nothing to see.

Hence, imagination 'understands and thinks the infinite by *not understanding and thinking* it'. Because of this, imagination 'knows that the infinite exists because it does not know it (νοεῖ δὲ ὡς ὑποστὰν ὅτι μὴ νοεῖ τὸ ἄπειρον)' (*In Eucl.* 285.15-18). The infinite is thus a very peculiar 'object' that cannot be thought because it is not a thing, which means that its essence is purely negative. As such, it cannot be known by itself: 'the infinite is altogether incomprehensible to knowledge (τὸ γὰρ ἄπειρον ὅλως ἐπιστήμῃ περιληπτὸν οὐκ ἔστιν)', because there is nothing to be known in the infinite except its very indefiniteness, which is not a thing (*In Eucl.* 285.26-286.1). In this sense, Proclus agrees with Plato and the Neoplatonic tradition.[11]

Therefore, the infinite is a *hypothetical* object that comes into being without being and derives from the productive power of imagination; as such it 'is' in 'being-other'. The infinite is not actual because to be *in actu* is to be limited, which the infinite precisely is not. The infinite is potential; it is the mere possibility of a geometrical object being extended, divided, or multiplied. Imagination, then, 'assumes the infinite not for the sake of the infinite but for the sake of the finite', i.e., for the sake and possibility of demonstration (πρὸς τὴν ἀπόδειξιν), where an always-finite line is extended as needed (286.1-4).

Imagination is thus indispensable for the constitution of geometrical objects in their being, construction, division, and extension, and constitutes the very geometrical matter into which mathematical forms are projected and where they are studied as extended by discursive reason with the aid of imagination, and subsequently are returned back to the simplicity of independent yet mutually communicative, thinkable forms.[12]

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[11] For Plato, matter cannot be properly known: it is conceived by 'illegitimate reasoning', ἁπτὸν λογισμῷ τινι νόθῳ, as though in a dream (*Tim.* 52b). For Proclus, this dream-like inadequate 'knowledge' of the infinite is precisely that which occurs of and in imagination. Cf. also Plotinus, *Enn.* ii 4(12).

[12] I am grateful to Philippe Hoffmann, Alain Lernoud, Dominic O'Meara, Bernard Vitrac, Alain

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