

Day-by-Day Schedule

Math 102, Spring 2019

Much Ado About Everything: The Mathematics of Leonhard Euler

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Lecture 1: Introduction to the course. Survey of Euler's vast mathematical output as a preview of what will follow.

Lecture 2: Al-Khwarizmi's solution of the quadratic (ca. 850). The solution of the cubic in 16th century Italy: del Ferro, Tartaglia, Cardano, Ferrari. Cardano's solution of the depressed cubic from the *Ars Magna* (1545).

Lecture 3: Six related issues, including how to depress a general polynomial and the "irreducible case" of the cubic that led Bombelli to introduce complex numbers (1570).

Lecture 4: Early 17th century mathematicians. Napier and Briggs and their logarithms (1615). Contributions of Descartes, Pascal, and Fermat.

Lecture 5: Newton's biography. His generalized binomial expansion and its use in the approximation of roots (1664).

Lecture 6: Newton's clever approximation of π via "inverse fluxions" (1669). Biographies of Leibniz and the Bernoulli brothers. The priority dispute over the creation of calculus.

Lecture 7: The brachistochrone challenge problem. Three divergence proofs of the harmonic series from Bernoulli's *Tractatus de seriebus infinitis* (1689). **PROJECT 1 DUE.**

Lecture 8: The first calculus textbook by l'Hospital (1696). Euler's biography. An initial look at his number theory, up through Pascal's lemma.

Lecture 9: Euler's first (induction) proof of the Little Fermat theorem and his factorization of $2^{32} + 1$, refuting a conjecture of Fermat (1736).

Lecture 10: Introduction to perfect numbers, including Euclid's proposition IX.36 from the *Elements* (ca 300 BCE). Euler's introduction of the σ function and its use to prove the Euclid-Euler theorem characterizing even perfect numbers (1750).

Lecture 11: Sylvester's proof that an odd perfect number must contain at least three different prime factors (1888). Amicable numbers and Euler's scheme for generating many new pairs (1750). **PROJECT 2 DUE.**

Lecture 12: Wrap up of number theory, including some famous unsolved problems. Euler's definition of "function" and his series expansions for exponentials from the *Introductio in analysin infinitorum* (1748).

Lecture 13: Euler's series for $\ln(1+x)$ and his clever calculation of natural logs. His use of infinitely small quantities to find differential formulas for $\ln x$, $\sin x$, and $\cos x$. Two proofs of Euler's identity, one via series and one via integral calculus.

MIDTERM EXAM

Lecture 14: Euler's evaluation of $\sqrt{-1}^{\sqrt{-1}}$. His proof of DeMoivre's theorem and determination of n^{th} roots of complex numbers (1749).

Lecture 15: Euler's ingenious approximation of π (1779). His original solution of the Basel problem, i.e., summing $1 + 1/4 + 1/9 + 1/16 + \dots$ (1735).

Lecture 16: Euler's general attack on the sums of reciprocals of powers by relating roots and coefficients of such series (1748). Some remarkable consequences.

Lecture 17: A few examples from his differential calculus text, including a surprising use of l'Hospital's rule to solve the Basel problem yet again (1755).

Lecture 18: One more amazing infinite series and then a look at Euler's integral calculus, including his evaluation of a difficult integral (1768).

Lecture 19: Euler's invention of analytic number theory with the product-sum formula and his proof that the sum of the reciprocals of the primes diverges (1737).

Lecture 20: Euler's introduction of the gamma function (1737). The proof of his quadrilateral theorem from Euclidean geometry (1750). **PROJECT 3 DUE.**

Lecture 21: Two proofs of Heron's formula, one by Newton (1677) and one by Euler (1750).

Lecture 22: Euler's *Algebra* and his factorization of real quartics (1770). Euler's brilliant foray into combinatorics with his theorem on derangements (1779).

Lecture 23: Euler's solution of the Bridges of Königsberg (1736). His 36-Officer problem (1782). His first partition theorem (1748).

Lecture 24: Mathematics after Euler: a *very* quick survey of Lagrange, Gauss, Cauchy, Weierstrass, Lebesgue, Ramanujan, etc. General wrap-up of the course. **PROJECT 4 DUE.**

FINAL EXAMINATION