MATH 121

Homework 12, due December 3

Part I

(1) Find the Jordan canonical form of the following operator, and record the change of basis you use.

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

(2) Compute the determinant of the following matrix in 3 different ways.

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

(3) Axler, page 245, Problem 12.

Part II

(4) Axler, page 246, Problem 21.

(5) We defined the sign of the permutation σ by writing σ as the composition of simple permutations switching only two elements (these simple permutations are called transpositions). If there are N transpositions in the decomposition of σ, then the sign of σ was defined by the formula $sign(\sigma) = (-1)^N$. Justify this procedure in two steps:

(a) Show that any permutation σ can be written as the composition of transpositions.

(b) Show that, although the number N of transpositions in the decomposition of σ is not uniquely determined by the permutation, the parity of N is determined by σ, so the sign $(-1)^N$ is well defined.