$-v \in W$. By the unique representation of 0 as the sum of a [vector in] U and a vector in W, we must have $v = 0$. Thus $U \cap W = \{0\}$, [completing] the proof in one direction.

[To prov]e the other direction, now suppose that $V = U + W$ and [$U \cap W = \{$]0$\}$. To prove that $V = U \oplus W$, suppose that

$$0 = u + w,$$

[where $u \in$] U and $w \in W$. To complete the proof, we need only show [that $u = w$] $= 0$ (by 1.8). The equation above implies that $u = -w \in W$. [Thus $u \in$] $U \cap W$, and hence $u = 0$. This, along with the equation above, [implies tha]t $w = 0$, completing the proof. ∎

1. Suppose a and b are real numbers, not both 0. Find real numbers c and d such that
$$1/(a + bi) = c + di.$$

2. Show that
$$\frac{-1 + \sqrt{3}i}{2}$$
is a cube root of 1 (meaning that its cube equals 1).

3. Prove that $-(-v) = v$ for every $v \in V$.

4. Prove that if $a \in \mathbf{F}$, $v \in V$, and $av = 0$, then $a = 0$ or $v = 0$.

5. For each of the following subsets of $\mathbf{F}^3$, determine whether it is a subspace of $\mathbf{F}^3$:

   (a) $\{(x_1, x_2, x_3) \in \mathbf{F}^3 : x_1 + 2x_2 + 3x_3 = 0\}$;

   (b) $\{(x_1, x_2, x_3) \in \mathbf{F}^3 : x_1 + 2x_2 + 3x_3 = 4\}$;

   (c) $\{(x_1, x_2, x_3) \in \mathbf{F}^3 : x_1 x_2 x_3 = 0\}$;

   (d) $\{(x_1, x_2, x_3) \in \mathbf{F}^3 : x_1 = 5x_3\}$.

6. Give an example of a nonempty subset U of $\mathbf{R}^2$ such that U is closed under addition and under taking additive inverses (meaning $-u \in U$ whenever $u \in U$), but U is not a subspace of $\mathbf{R}^2$.

7. Give an example of a nonempty subset U of $\mathbf{R}^2$ such that U is closed under scalar multiplication, but U is not a subspace of $\mathbf{R}^2$.

8. Prove that the intersection of any collection of subspaces of V is a subspace of V.

9. Prove that the union of two subspaces of V is a subspace of V if and only if one of the subspaces is contained in the other.

10. Suppose that U is a subspace of V. What is $U + U$?

11. Is the operation of addition on the subspaces of V commutative? Associative? (In other words, if U_1, U_2, U_3 are subspaces of V, is $U_1 + U_2 = U_2 + U_1$? Is $(U_1 + U_2) + U_3 = U_1 + (U_2 + U_3)$?)