SYLLABUS FOR MATHEMATICS 121

Where and when: Fall 2014, MWF 1–2pm in Science Center 507

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Prerequisites: Math 21b or equivalent. Should not ordinarily be taken in addition to math 23a, 25a, or 55a.

Textbook: *Linear Algebra Done Right*, by Sheldon Axler.

Last night as I was walking along a country lane I came upon a man searching the ground beneath a street light. I asked him, "What are you looking for?"

He replied, "My contact lens'.'

I asked, "You lost your contact lens under this street light?"

"No, I lost it in the field over there."

"Then why are you looking for it under this street light?"

"Because this is the only light I've got!"

Linear algebra is to a mathematician much like the street lamp is to the man searching for his contact lens in the bad joke above — we're not generally trying to solve a problem in linear algebra, but it's often the best tool we've got. Linear algebra is a very beautiful theory that is extremely well understood and applicable in a wide variety of situations both within pure mathematics and in other fields. That said, this is a course in theoretical linear algebra. We will touch on some interesting applications in applied math and computer science, but the main focus will be on learning the theory. If you want to learn linear algebra as applied to another discipline and are not interested in linear algebra as a topic in its own right, you may want to take a look at an analogous course in a different department.

Goals

Goal 1: To cover the main topics in theoretical linear algebra.

Goal 2: To expose you to the language of mathematics, i.e. to develop your skill in writing proofs.

Pure mathematics is indeed a language, and it requires practice in order to become fluent. Experience thinking about a problem abstractly and formulating a solution rigorously is useful outside of pure math as well, and is one of the primary goals of this course.

Course work and expectations

There will be weekly homework assignments. In addition, students are expected to attend lecture, attend sections, and read the text. There will be one in-class midterm and a three-hour final examination.

Some course policies:

- There will be a homework assignment due at the beginning of class every Wednesday. Late homework will generally not be accepted. Exceptions may be made in exceptional circumstances, and should be arranged in advance. Each student's lowest two homework grades will be dropped.
- You are encouraged to work with your fellow students when you are solving the homework problems. However, you are expected to write up the solutions on your own. To be clear: finding a solution to a problem and presenting it on paper in a rigorous way are two separate steps; the former may be done in a group, and the latter must be done alone. (It may take as much time to write up as to find the solutions.) Please list the names of all collaborators on any work that you hand in.

  Of course, no collaboration is permitted on exams.
- You are expected to come to all of the lectures. That said, I am not going to take attendance or anything — you're adults. I encourage you to participate in the lectures and ask as many questions as you like — American mathematical culture has the wonderful quirk that anyone is allowed to demand justification of any statement, at any time (within reason) during a talk or a lecture.

  It is important to attend problem sections led by the CA.

Grading

Your grade will be based on the quality of your submitted homework assignments as well as your performance on the midterm and final examination, according to the following weights:

- 50% homework
- 20% midterm
- 30% final

There will be an in-class midterm on October 15, and a Final exam at the assigned time during Finals week.

The grades will not be curved, although letter grade cutoffs will not be decided in advance. Your lowest homework score will be dropped. On each homework assignment, 95% of your score is earned by having correct solutions, and 5% by the quality of the exposition. So in order to get a perfect score, your work needs to be legibly written in complete sentences and correct English (with mathematical notations taken into account). As mentioned above, you are encouraged to work on the problems with other

people, but you must write up the solutions on your own. Please list any collaborators on any work that you hand in.

Since exams are timed and are done individually, they will only be graded for correctness.

Rough course schedule

Weeks 1–2: vector spaces, subspaces, and bases.

Weeks 3–4: linear transformations and matrices; application to differential equations.

Week 5: elementary matrix operations and solving systems of linear equations.

Weeks 6–7: determinants.

Weeks 8–9: eigenvalues, eigenvectors, and diagonalization; application to computer science.

Week 10: Jordan canonical form and rational canonical form.

Weeks 11-13: Inner product spaces, duality, tensor products, and other topics.

Advice to students

- Read and understand the text before doing the homework assignment. This will generally save you time in the long run, and there may be things in the text that get skipped in lecture.
- Make time to write up your solutions. Finding the solutions to the homework problems is one thing; figuring out how to explain it to the grader is another thing entirely. Proofs that are hastily jotted down during a problem session are generally illegible.
- You will have questions. Your CA and I will have office hours. Take advantage of them.
- Go to the discussion sections. Your CA will have interesting and useful things to tell you.