

## What is Math 124?

NUMBER THEORY is everywhere in mathematics! Few mathematical subjects can do entirely without it, and almost every mathematical subject can find its application in problems of number theory. Modern number theory has important contact with many aspects of geometry, topology, combinatorics, high-speed computation & data collection, and, of course, with physics.

Math 124 is the “introductory course” for this subject, its principal aim being to make students *utterly* familiar with the grand classical work of Gauss, and others that have laid the foundations of modern research in this area. It is surprising how few technical prerequisites it demands! Our subject folds into one and the same package, the continuous methods of calculus, the discrete methods of algebra, and offers us the fun of making of basic, illuminating, computations.

Here are some of the landmarks of our syllabus.

1. The basic machinery of congruence.
2. Unique Factorization (in various rings) and non-Unique factorization (in other rings) and Applications. These applications will range from:
  - Theorems regarding the distribution of prime numbers in the set of all positive numbers.

*Note: The famous Riemann Hypothesis—still unproved—is a very precise statement about this distribution. We are not going to cover that in Math 124, of course, but see the draft of a book that I am currently writing with William Stein “What is Riemann’s Hypothesis?” <http://wstein.org/rh/>*

- An introduction to the theory of binary quadratic forms.

*The material discussed so far is covered in the first four chapters of Ireland and Rosen, and in the corresponding chapters of Davenport, all this leading seamlessly to:*

3. Quadratic reciprocity, and quadratic Gauss sums.

*This is material covered in chapters five and six of Ireland and Rosen, and in the corresponding chapters of Davenport, leading to:*

4. The study of class numbers, ideal class groups, Pell’s equation, and continued fractions.

This, then, is the backbone of the course. If time permits we will study topics in the later chapters of Ireland and Rosen. For example, perhaps, all of Chapter 10, or selections from Chapter 17.