QR 26: Choice and Chance
The Mathematics of Decision Making

Unit II Exercises

II.A (1) Let A be a set of five automobile brand names. Display your weak preference structure P on A using a 5-by-5 array. Check that this relation P has the two properties needed to be considered a preference relation. Indicate where strict preference and indifference hold.
(2) Find a value function v on A that represents your preferences. Find another one w that also represents your preferences but without having $v(x)=aw(x)+b$ for all x in A.

(3) Suppose you and your neighbor each have two car garages. Show that you can find a v and w each representing your preferences such that the average value of the cars in your garage is greater than for his garage when counted according to v, but his garage has greater average value according to w.

II.B Revisit the choice among six projects from the Unit II Activity as given again below. Calculate the Net Present Value (NPV) of each project assuming a discount rate of 10%. Compare this ranking to the one obtained by calculating each project's Internal Rate of Return (IRR). Explain why the IRR is a more reasonable criterion for accept-or-reject decisions than for comparing two or more projects.

	Cash Flow					
Year	A	B	C	D	E	F
0	-10m	-10m	-10m	-1m	-16m	-16m
1	+5m	+5m	+2m	+0.5m	+16m	+3.2m
2	+5m	+5m	+8m	+0.5m	+5m	+19.2m
3	0	+5m	+5m	+0.5m	0	0
4	0	+5m	+5m	+0.5m	0	0

II.C A firm is considering whether to manufacture a number of products, each of which has a two year life. There are only two attributes of importance for each product: X, cash flow in year 1, and Y, cash flow in year 2. These cash flows may be forecast with perfect accuracy. Suppose the firms preferences has weak preferences R and indifference relation I satisfying:

(i) X and Y are mutually preferentially independent.
(ii) $(x, y)\ R\ (x', y)$ for all $y \Leftrightarrow x \geq x'$
(iii) $(x, y)\ R\ (x, y')$ for all $x \Leftrightarrow y \geq y'$
(iv) $(100, 400)\ I\ (200, 300)$ and $(0, 600)\ I\ (100, 200)$.

Consider the following pairs of products. In each case, if only one can be manufactured, which should be? Give reasons. [F]

(1) (0, 100) or (100, 100)
(2) (0, 400) or (200, 200)
(3) (100, 500) or (200, 300)
(4) (0, 500) or (200, 400)?

II.D Winnie the Pooh consumes honey and condensed milk. He always prefers more honey, and he also prefers to consume more condensed milk up to some amount c* that is independent of the amount of honey consumed, but above c*, he prefers less condensed milk. If (h, c) denotes the act of consuming h jars of honey and c cans of condensed milk, we can say that his strict preferences P satisfy

(h' , c) P (h", c) if h' > h"
(h , c') P (h , c") if c* ≥ c' > c"
(h , c") P (h , c') of c' > c" ≥ c*

Sketch what you would expect the indifference curves to look like in the plane and briefly justify your answer. Could Pooh have an additive value function? Any ideas about how to construct one? [F]

II.E Use the mid-value technique and a spreadsheet to graph your own desirability function d on a scale of 0 to 10 for money between $0 and $1,000,000. Start by deciding the level of dollars x* for which going from $0 to $x* is just as desirable as an increase from $x* to $1,000, 000. (Try thinking of a single amount of work or of some other independent item you would swap both for going from $0 to $x* or for going from $x* to $1,000,000.) Describe and interpret some of the qualitative features of your desirability function d. For example, is it increasing? Concave (like a bowl face down) or convex (like a bowl opening upwards)? Continuous? Explain how you think your function would compare to one drawn by an older relative such as a parent?

II.F *Choice of a Graduate Program*

If you are thinking of attending a professional school (law, medicine, business) after college, you are probably trying to decide which school is best for you. (If you aren't thinking of going to a professional school, pretend for a minute that you are – choose one of the three fields listed above.)

i. To avoid anchoring, write down three or four of the most important objectives you have in this problem. How might you choose scales to evaluate these objectives? What sort of scales would they be? What units might you use (if they are interval scales)?

Turn your web-browser to http://www.usnews.com/usnews/edu/beyond/bcrank.htm. In the left-hand margin of the page, you will see links to "compare business schools" or "compare law schools" or "compare medical schools". Click on the icon that applies to you. You will then be asked to enter four graduate programs of your choice. Enter at least three, and click on the "compare schools" button at the bottom of the page. Answer the following questions about what you see.

ii. What general categories of objectives has U.S. News selected in preparing this report? Prepare a list of four or five. Don't consider specific sub-objectives like "% employed in manufacturing" – provide a more general grouping, more like your list of objectives.

iii. For each objective of your own, select a proxy variable from the data listed that you think best represents that objective. If your objective is not at all reflected in the objectives of U.S. News, you can add information from other sources and from your personal knowledge. Make a consequence table, as we did in class. Evaluate the four graduate programs by first doing a purely ordinal analysis, and then making tradeoffs as necessary. Come up with a ranking of the schools.

Now, go back to http://www.usnews.com/usnews/edu/beyond/bcrank.htm. Look up the rankings for graduate schools in your area of interest.

iv. In what order are your schools listed? Is this different from your analysis? How do you think U.S. News weights its objectives, and how does this compare to your analysis? Do you think they've left out anything?

v. When you have finished (iv) go back to http://www.usnews.com/usnews/edu/beyond/bcrank.htm and look at "methodology" for the professional-school of your choice. What sort of weighting is this (linear, non-linear)? List examples of all the different types (nominal, ordinal, interval, ratio) of scales they make use of. Make a criticism of the paragraph on the "Reputation" component of the ranking. Does this component make use of a meaningful statistic?

Mathematical Challenge #1: Graphically, with Excel, or otherwise, find the internal rate of return for the project below and comment on what is so curious about this example.

Year	0	1	2	3

Cash Flow	-25	125	-204	108

Mathematical Challenge #2: A function f from a set A to a set B is just a special kind of relation. What kind? If A and B are finite, you can represent a function using a finite array as we did for preferences. Give some examples. What if one of A or B is infinite? If both A and B are the positive integers, does such a representation make sense? What if both A and B are the real line?