QR 26: Choice and Chance
The Mathematics of Decision Making

Midterm Exam Review

Unit I

1. Perspectives on decision-making: *descriptive, normative, and prescriptive*.
1. The *PrOACT* method of analysis: this should be your first step in breaking down any decision problem. Generally, it shouldn't be the last. *Problem, objective, alternatives, consequences, tradeoffs*. *Consequence table* and *conditional rankings table.*
2. Psychological traps for DM under certainty: *anchoring trap, status quo bias, sunk cost trap, confirming-evidence trap, framing trap* (for a review, see ch. 10 of Smart Choices)
3. Measurement scales (describing consequences): *nominal, ordinal, interval, ratio* scales (see Growney). Construction of scales and *proxy variables*.
4. *Binary relations* (see handout on set theory) and *preference structures* (= a binary relation that is *transitive* and *complete*). *Strong preference (R), indifference (I*), and *weak preference (P)* relations, their interpretations, and some of their properties. Strong preference and indifference together form a preference structure on a set; weak preference forms a preference structure on a set all by itself. How to represent preference structures using *tables*, and check for inconsistencies. *Independence of irrelevant alternatives (IIA)* and how to detect violations.
5. *Value functions*: in general, any function *v* that satisfies the condition $(x\ P\ y) \Leftrightarrow v(x) \geq v(y)$. Uniqueness up to an increasing transformation.

Unit II

1. *Consequence tables* (cf. Unit 1); usually, rows = objectives and columns = alternatives. *Conditional rankings table* for a first-cut analysis. The method of *equal swaps*. *Dominance* and *weak dominance* as means of eliminating columns (alternatives) and *irrelevance* as a means of eliminating rows (objectives).
2. Introduction to *multi-attribute value theory (MAVT)*. Representation of multi-attribute "bundles" in "*act space*" -- n-dimensional space. The new condition for the multi-attribute value function is: $(x_1, x_2, \ldots, x_n)\ P\ (y_1, y_2, \ldots, y_n) \Leftrightarrow v(x_1, x_2, \ldots, x_n) \geq v(y_1, y_2, \ldots, y_n)$. Of the set of alternatives, only the non-dominated region is worth considering for a final choice. Among these choices, the one with the highest value function should be chosen.
3. *Additive value functions (AVF's)*: value functions of the form $v(x_1, x_2, \ldots, x_n) = v_1(x_1) + v_2(x_2) + \ldots + v_n(x_n)$. If you have an AVF, then you can say that the attributes $(x_1, x_2, \ldots, x_n)$ are *preferentially independent*. Intuitively, preferential independence means that our preference ordering on some of the attributes does not depend on the levels of the other attributes. A very special type of AVF is *a linear additive value function (LAVF).* In this situation, the amount contributed to the value function by a given change in a given variable is a constant over the entire range of the variable; thus, the function has the form $v(x_1, x_2, \ldots, x_n) = w_1 * x_1 + w_2 * x_2 + \ldots + w_n * x_n$.
4. Tradeoffs between present and future: *discounting*. A dollar today is not worth a dollar tomorrow. *Discount rate = r; discount factor = d; d = 1/(1+r).* A *cash flow* is a series of payoffs (or payouts) C_0, C_1, … where the subscript is the year of the payoff. Negative values of C_i imply a payment made in the *i*-th year; year 0 is the present. The idea is that you "undo" the effects of interest; to convert a payoff in year *n* to a payoff in year 0 (the present), you multiply C_n by d^n, where *d* is the discount factor = $1/(1+r)$ (this gives you the *present value* of C_n). The sum of the present values of all the future payoffs give the *net present value (NPV)* of the cash

flow. *Internal rate of return (IRR)* is the value of the discount rate *r* such that the NPV of the cash flow is equal to zero. To find *r*, first find *d* by solving the equation $C_0 + C_1 * d + \ldots + C_n * d^n = 0$. This model of discounting assumes: (1) stationarity of discount rate, and (2) preferential independence between cash flows in different years.

5. Population dynamics. You should know what a difference equation is (whale problem). Doubling time of a quantity.

<u>Unit III</u>

1. *Optimization*: to make the most of something. *Objective function* ~ value function (the thing to maximize). *Choice variables*: the variables in a problem that we can choose values for. A *feasible set* is the set of possible ("feasible") values for the choice variables; this size and shape of this set is determined by the *constraints* of the problem. Linear programming: *linear* means all constraints and objective functions are of the general form $c_1 * x_1 + c_2 * x_2 + \ldots + c_n * x_n$., where the c_i's are constants and the x_i's are variables; there are no squares, roots, logarithms, sines, etc., of the variables. The objective function corresponds to a LAVF in this situation.
2. *Corner principle*: if there is a solution to a linear program, then there will be a solution on a corner of the feasible set. For a problem with finitely many linear constraints, there will be finitely many corners to the feasible set; find the coordinates of the corners, and check these in the objective function; the one with the highest value is the optimal solution.
3. *Slack v. binding constraints*: If a constraint is binding, that means that that constraint is one of the constraints that is actually preventing the value of the objective function from being higher in this situation; thus, you would like this constraint to go away and thereby leave you with a point that is higher on your objective function. *A slack constraint* is a constraint that could be deleted from a situation without affecting the optimal solution. The maximum amount you would be willing to pay to relax the constraint by one unit is called the *shadow price* of the constraint. However, you cannot relax the constraint indefinitely and expect it to remain binding – at some point, the other constraints will take over, and the optimal solution will no longer touch the other constraint. This is the point of *sensitivity analysis* – you must see how big a deviation in your constraint the linear program will tolerate before the optimal solution switches to another vertex on the frontier.
4. *LP Assumptions*: (1) linearity; (2) continuous divisibility of the choice variables.