QR 26: Choice and Chance
The Mathematics of Decision Making

http://www.fas.harvard.edu/~qr26

Purpose

This course develops mathematical ideas that can help individuals make rational choices. We study both decisions whose results are predictable and those made under uncertainty, including cases designed for professional school classes. Topics range from optimization under constraints to Bayesian probability theory, and from iterated dynamical systems to empirical surprises concerning how people make estimates and bets in practice.

Prerequisites

The only prerequisities are high school algebra and a willingness to think hard.

Faculty

Daniel Goroff	Howard Raiffa
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Classes

Tuesdays and Thursdays from 10:00-11:30, initially in Science Center 112.

Sections

Wednesdays from 4:00 to 5:00 in Science Center 101b.
Teaching Fellow: Matthew Leingang (leingang@math.harvard.edu)
Office Hours: Tuesdays and Thursdays from 2:00 to 3:00 in Science Center 505g

Materials

Text: *Smart Choices* by Hammond, Keeney, and Raiffa. Harvard Business School Press 1999.
Sourcebook: Available at the HPPS window in the Science Center Basement.
Recommended: *Judgment in Managerial Decision Making* by Bazerman. Wiley 1998.

Software: *Microsoft Excel* available in Science Center Labs or in a student edition from TPC.
Excel Add-On: *TreePlan* software available for course use for the Mac on the FAS server under courseware, or for Windows through the course webpage,www.fas.harvard.edu/~qr26.

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Outline

The six units progress so as to emphasize how ideas about decisions with known outcomes generalize to situations where there is uncertainty. Concentrating on prescriptive theory concerning how you can learn to make better choices, we will also distinguish between descriptive theory concerning real behavior and normative theory concerning ideal behavior. Trying in this way to understand what is consistent, rational, and quantifiable about decision-making can also help us understand better what is not.

Unit I: *The Logic of Preferences*

Formulating personal decision problems. Formalizing the mathematical properties, relations, and conditions for describing how individuals order alternatives in practice and in theory.

Unit II: *Tradeoffs Under Certainty*

How equal swapping and weighted scoring schemes can help sort through choices among items with multiple attributes, including the special case when payoffs occur in a stream over time.

Unit III: *Optimization and Mathematical Programming*

Geometrical and spreadsheet-based techniques for maximizing a quantity that depends on constrained variables, especially the useful and highly structured case of linear programming.

Unit IV: *Judgement and Decisions Under Uncertainty*

Probability as a way individuals can order uncertain alternatives. How to separate attitudes towards risk from attitudes towards reward. Simple decision trees. Behavioral anomalies.

Unit V: *Inference, the Value of Information, and Sequential Choice*

Deciding whether to act now or collect more information. Revising Bayesian probabilities. Solving compound and evolving decision trees. Common mistakes, biases, and anomalies.

Unit VI: *Perspectives and Extensions*

How ideas developed for personal decision problems relate to game theory, negotiation theory, portfolio theory, statistics, voting theory, group decision theory, social choice theory, etc.

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Coursework

Each of the six units will last about two weeks. A study guide will be distributed at the beginning of each unit indicating readings, lecture outlines, and three types of assignments:

(1) *Activities* for the beginning of the Unit, usually survey questions with no right or wrong answers. Worth 4 points each.
(2) *Exercises* for the middle of the Unit, usually one problem set that develops thinking, calculating, and computer skills. Worth 12 points each.
(3) *Projects* at the end of the Unit, usually a case study for discussion. Worth 4 points for the one or two page response paper you bring to class, and 4 points for in-class participation.

As an example of a project, you will be asked during the first Unit to identify two or three personal decisions faced by you or someone close to you. In a diary, you will revisit at least one such decision throughout the semester in view of what we are learning, and submit your cumulative analysis of this personal decision for credit during the final Unit.

Once during the semester, your unit project can be to work with a group of three or four to record, analyze, and report on the survey results from that unit's opening activity.

Termwork as above is potentially worth up to 144 points. Assignments handed in past the due dates indicated on the Unit Study Guides will not be accepted. You are encouraged to work in groups, but you must write up the finished product in your own words and credit your sources.

A *Midterm* worth 100 points is tentatively scheduled for in class on Thursday, March 23, 2000.

The *Final Examination* will also be worth 144 points, so that it and the termwork each count for approximately three-eighths of your grade, while the midterm counts for a quarter.

Sample Questions

By the end of QR 26, you should feel confident answering questions like these:

If you know that a family has two children, one of whom is a son, what is the probability the other child is a daughter? (If you think the answer is one half, think some more.)

A friend has a streak of heads tossing a fair coin and wants to bet heavily that the next toss must finally turn up tails. What would you advise?

A friend choosing among three items says that he has good reasons to prefer alternative *x* to *y*, and *y* to *z*, but *z* to *x*. What rationalizations could account for this behavior and what would you advise? From what would you want to protect your friend?

More QR26 Sample Questions

What about the following data convinced the U.S. Congress not to pass a proposal for requiring couples applying for marriage licenses to take the HIV blood test: about 0.3% of the U.S. population carries the HIV virus; a person with the HIV virus has a 95% chance of testing positive; and an HIV virus free person has a 4% chance to test positive?   [AC]

The Yankees are playing the Dodgers in a world series. The Yankees win each game with probability 0.6. What is the probability that the Yankees win the series? (The series is won by the first team to win four games.)   [GS]

How could you calculate a transportation plan for minimizing the total cost of moving goods from several warehouses with given inventories to several outlets with given orders?

Each of two people of comparable age and health has a revolver. The first gun has bullets in three of its six chambers; the second has a bullet in only one of its six chambers. Each person is going to spin the cylinder, put his gun to his head, and shoot. Knowing nothing else about the situation, you may remove one bullet from one gun beforehand. Which bullet do you take? Compare this to the problem of a doctor who has the resources to treat only one of two patients, both of whom are seriously ill but with different survival chances.   [F]

There is lively debate in Massachusetts over whether to add a new runway at Logan or greatly expand the airport in Worcester. If you were advising a state official on non-political aspects of this decision, how would you begin formulating the problem? What data would you need?

What kinds of biases, inconsistencies, and irrational behaviors do experimenters routinely observe when people make decisions in practice? How can you avoid or use these traps?

How can you think better about multiple and conflicting objectives in decisions such as choosing a job, buying a car, renting an apartment, etc.?

How can you explore and summarize your own basic attitudes toward risk in order to help you handle complex risky choices?

In cases concerning medical, drug, or pollution testing, for example, when should you act under present imperfect information as opposed to gathering additional information at some cost that might change your mind about key uncertainties?

In the purse snatching case *People v. Collins*, a couple seen fleeing the scene in Los Angeles were described as a black man with a beard and a blond girl with a ponytail driving a partly yellow car. Malcom and Janet Collins were arrested because they and their car matched this description. A mathematician testified that the chances of a randomly chosen couple having

these characteristics was smaller than one in twelve million, and they were convicted. How might you, as an expert witness, reach such a conclusion? What calculation did lawyers present on appeal that made the California Supreme Court overturn the lower court's initial guilty verdict nevertheless?   [GS]