

# Quantitative Reasoning 28: The Magic of Numbers

## Homework 18

Assigned on March 23

**Due at 5:00 p.m. March 25**

Please submit problem sets to the boxes outside the Math Department's main office, on the third floor of the Science Center (Room 325).

### Reading:

Gross-Harris, Chapter 17

### Problems:

Please explain your reasoning and show your work.

Remember that when working  $(\text{mod } n)$ , your final answer should be a symbol in the range  $0, 1, \dots, n - 1$ . Also recall that, when asked to do division in modular arithmetic, the answer could be “does not exist.”

1. The goal of this problem is to find  $5/17 \pmod{31}$ . Note that 31 is prime, and thus we know that a solution exists.
  - (a) Use the Euclidean Algorithm to find integers  $x$  and  $y$  such that  $17x + 31y = 1$ .
  - (b) Using part (a), what is  $1/17 \pmod{31}$ ?
  - (c) Now multiply your answer to part (b) by 5 to find  $5/17 \pmod{31}$ .
2. Do the following divisions.
  - (a)  $7/10 \pmod{40}$ .
  - (b)  $9/23 \pmod{40}$ .
  - (c)  $10/2 \pmod{31}$ .
  - (d)  $5/16 \pmod{17}$ .
  - (e)  $10/2 \pmod{20}$ .
  - (f)  $1/9 \pmod{44}$ .