Quantitative Reasoning 28:
The Magic of Numbers

Homework 27

Assigned on April 25
Due at 5:00 p.m. April 27

Please submit problem sets to the boxes outside the Math Department's main office, on the third floor of the Science Center (Room 325).

Reading:

Gross-Harris, Chapters 22 and 23

Problems:

Please explain your reasoning and show your work.

1. Eve listens in on a communication between Bob and Amanda. She knows that Bob transmitted to Amanda $n = 2047$, $k = 125$. Amanda responded with the number 2. What was Amanda's message? (Computational hint: $n = 2047$ is close to a power of 2. Use this to your advantage.)

2. A number b between 1 and $n - 1$ is called a *witness* for the fact that n is composite, or simply a witness for n, if $b^{n-1} \not\equiv 1 \pmod{n}$. (As was discussed in today's lecture: if n were prime, Fermat's little theorem would say that $b^{n-1} \equiv 1 \pmod{n}$. So n has to be composite if it has any witness).

   (a) Show that 3 fails to be a witness for 91.

   (b) Show that 2 is a witness for 255.

3. Find two witnesses to the fact that 121 is composite.