Quantitative Reasoning 28:
The Magic of Numbers

Homework 28

Assigned on April 27
Due at 5:00 p.m. April 29

Please submit problem sets to the boxes outside the Math Department's main office, on the third floor of the Science Center (Room 325).

Reading:

Gross-Harris, Chapter 23

Problems:

Please explain your reasoning and show your work.

1. For each number in arithmetic (mod 15), compute its square.

   (a) Which numbers are squares?

   (b) How many square roots does each number have?

2. Suppose you know that 341 is a pseudoprime to base 2; that is, $2^{340} \equiv 1$ (mod 341) (this is, in fact, true).

   (a) Compute 2^{170} (mod 341). Hint: what is 2^{10} (mod 341)?

   (b) Keeping in mind that $2^{340} = \left(2^{170}\right)^2$, what does this tell you about the primality of 341?

   (c) Compute 2^{85} (mod 341).

   (d) Keeping in mind that $2^{170} = \left(2^{85}\right)^2$, what does this tell you about the primality of 341?

3. On the last homework, we saw that $3^{90} \equiv 1$ (mod 91). Thus the Fermat test didn't unmask 91 as a composite. Starting from $3^{90} \equiv 1$ (mod 91), apply the Miller-Rabin test and report what it reveals about the primality of 91.