Homework 10 Solutions

1. (a) **Find the greatest common divisor of 161 and 205.**

    Using the Euclidean algorithm gives:

    $$\begin{aligned}
    205 &= 161 + 44 \\
    161 &= 3 \cdot 44 + 29 \\
    44 &= 29 + 15 \\
    15 &= 14 + \boxed{1}.
    \end{aligned}$$

   (b) **Find whole numbers x and y so that $161\mathbf{x} + 205\mathbf{y} = 1$.**

    Running the Euclidean algorithm backwards gives:

    $$\begin{aligned}
    1 &= 15 - 14 \\
    &= 15 - (29 - 15) = -29 + 2 \cdot 15 \\
    &= -29 + 2(44 - 29) = 2 \cdot 44 - 3 \cdot 29 \\
    &= 2 \cdot 44 - 3(161 - 3 \cdot 44) = -3 \cdot 161 + 11 \cdot 44 \\
    &= -3 \cdot 161 + 11(205 - 161) = -14 \cdot 161 + 11 \cdot 205.
    \end{aligned}$$

    Thus, $\boxed{x = -14,\ y = 11}$ is a solution (although not the only one, of course).

2. (a) **Find the greatest common divisor of 111 and 234.**

    Using the Euclidean Algorithm:

    $$\begin{aligned}
    234 &= 2 \cdot 111 + 12 \\
    111 &= 9 \cdot 12 + \boxed{3} \\
    12 &= 4 \cdot 3 + 0
    \end{aligned}$$

   (b) **Find whole numbers x and y so that $111x + 234y = 6$.**

    Running the Euclidean algorithm backwards gives:

    $$\begin{aligned}
    3 &= 111 - 9 \cdot 12 \\
    &= 111 - 9(234 - 2 \cdot 111) = 19 \cdot 111 - 9 \cdot 234
    \end{aligned}$$

    Multiplying this equation by 2 gives

    $$6 = 38 \cdot 111 - 18 \cdot 234.$$

    So we may take $\boxed{x = 38,\ y = -18}$.

3. (a) **Find the greatest common divisor of 186 and 270.**

    By the Euclidean Algorithm:

    $$\begin{aligned}
    270 &= 186 + 84 \\
    186 &= 2 \cdot 84 + 18 \\
    84 &= 4 \cdot 18 + 12 \\
    18 &= 12 + \boxed{6} \\
    12 &= 2 \cdot 6 + 0
    \end{aligned}$$

   (b) **For which whole numbers m is it possible to find whole numbers x and y so that $186x + 270y = m$?**

    We know from lecture (or the book), that we can solve this equation exactly when m is a multiple of the gcd, that is, whenever $\boxed{m \text{ is a multiple of } 6}$.