

Homework 12 Solutions

1. **Determine if each of the following numbers is prime or composite. Remember, you only need to check for factors up to the square root of the number.**

(a) 681;

The square root of 681 is between 26 ($26^2 = 676$) and 27 ($27^2 = 729$). If we begin checking the primes less than 26 to see if they divide 681, we quickly see that 3 does ($681 = 3 \cdot 227$). Thus 681 is composite.

(b) 739;

The square root of 739 is between 27 ($27^2 = 729$) and 28 ($28^2 = 784$). If we check the primes less than 27 to see if they divide 739, we see that none of them do. Thus 739 is prime.

(c) 729;

We saw above that $729 = 27^2$. Hence 729 is composite.

(d) 683.

The square root of 683 is between 26 ($26^2 = 676$) and 27 ($27^2 = 729$). If we check the primes less than 26 to see if they divide 683, we see that none of them do. Thus 683 is prime.

2. **Which of the numbers between 300 and 325 are prime? (Hint: you can test them simultaneously using the Sieve of Eratosthenes.)**

We won't write out the final form of the sieve (it's probably not too helpful, and definitely a pain to type). After crossing out the multiples of all the primes up to 17 (since $19^2 = 361$, we can stop at 17), we're left with 307, 311, 313, and 317. Thus, these are the primes between 300 and 325.

3. **Two prime numbers that differ by 2 are called "twin primes." Use the Sieve of Eratosthenes to find a pair of twin primes greater than 200. (Note: As mentioned in lecture, a famous unsolved problem in mathematics is whether or not there are infinitely many pairs of twin primes.)**

Again, we won't write out the sieve. The first pair greater than 200 is 227 and 229, and the next is 239 and 241. Some people observed that 311 and 313 from the previous problem were twin primes greater than 200, and thus they qualify too.