Homework 13 Solutions

1. **Factor the following into prime numbers:**

   (a) $\binom{20}{7}$;

   Recall that
   $$\binom{20}{7} = \frac{20!}{13! \cdot 7!} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}.$$
   The denominator is $7 \cdot 2 \cdot 6 \cdot 3 \cdot 5 \cdot 4 = 14 \cdot 18 \cdot 20$. Cancelling these out gives $19 \cdot 17 \cdot 16 \cdot 15$. Factoring these numbers gives $\boxed{2^4 \cdot 3 \cdot 5 \cdot 17 \cdot 19}$.

   (b) 7007;

   It's obvious that $7007 = 7 \cdot 1001$. If we try dividing 1001 by primes, we see that $1001 = 7 \cdot 143$. Finally, $143 = 11 \cdot 13$, and thus $7007 = \boxed{7^2 \cdot 11 \cdot 13}$.

   (c) 1991;

   If we start dividing by primes, we see that $1991 = 11 \cdot 181$. Trying to divide 181 by primes up to 13 (since $14^2 = 196$), we see that 181 is prime. Thus $1991 = \boxed{11 \cdot 181}$.

2. **Today in lecture, Prof. Mazur constructed a table showing the remainder of $A \times B$ divided by 3 given the remainders of both A and B when divided by 3. Construct an analogous table for 5. (That is, given the remainder of A and B divided by 5, determine the remainder of $A \times B$ divided by 5, for all possible cases.)**

   For example, consider the case when $A = 5n + 3$ and $B = 5m + 2$. Then
   $$\begin{aligned} A \times B =& 25nm + 10n + 15m + 6 \\ =& 5(5nm + 2n + 3m + 1) + 1, \end{aligned}$$
   and we see that the entry for 2 and 3 (in either order) is 1. Note that we now know that this is the same as computing $2 \cdot 3 \equiv 1 \pmod 5$. The others can be done similarly, and the resulting table is

	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1