Homework 17 Solutions

1. **Do the following computations in the given modulus.**

   (a) $6 - 4 \pmod 7$.

   We compute that $6 - 4 = \boxed{2}$, and since this is already is the range 0 to 6, we're done.

   (b) $80 + 21 \pmod{101}$.

   We see that $80 + 21 = 101 \equiv \boxed{0} \pmod{101}$, since 101 obviously has remainder 0 when divided by 101.

   (c) $3 - 12 \pmod{15}$.

   We have $3 - 12 = -9 \equiv \boxed{6} \pmod{15}$.

   (d) $456 \cdot 450 \pmod{457}$.

   For this problem, it's helpful to observe that $456 \equiv -1 \pmod{457}$ and $450 \equiv -7 \pmod{457}$. Thus, $456 \cdot 450 \equiv -1 \cdot -7 \equiv \boxed{7} \pmod{457}$.

2. **Do the following divisions by trial and error. (Note that you don't need to write out the whole multiplication table; just try the different possibilities. Eventually, we'll develop a more systematic approach.)**

   (a) $3/5 \pmod{13}$ **(Remember that, by definition, this is the number which, when multiplied by 5, gives you 3 in** $\pmod{13}$**.)**

   We want a number which, when multiplied by 5, is 3 more than a multiple of 13. So we should look at numbers of the form $3 + 13x$ until we find one which is divisible by 5. The first few such numbers are 3, 16, 29, 42, and 55. We stop here because $55 = 5 \cdot 11$, and thus $11 \cdot 5 = 55 \equiv 3 \pmod{13}$. As mentioned in the problem, this means that $3/5 \equiv \boxed{11} \pmod{13}$. (Note that 13 is prime, and thus we don't have to worry about this division not being defined.)

   (b) $8/11 \pmod{17}$.

   We proceed in the same way as the previous problem. The first few numbers of the form $8+17x$ are 8, 25, 42, 59, 76, 93, and 110. Since $110 = 10 \cdot 11$, we conclude that $8/11 \equiv \boxed{10} \pmod{17}$.

3. **Compute the following.**

   (a) $5^{491} \pmod{24}$.

   If we begin the process of successive squaring, we immediately see that $5^2 = 25 \equiv 1 \pmod{24}$. Thus

   $$5^{491} = \left(5^2\right)^{245} \cdot 5 \equiv 1^{245} \cdot 5 \equiv \boxed{5} \pmod{24}.$$

   (b) $109^{10} \pmod{37}$.

   We begin by computing $109 \equiv 35 \equiv -2 \pmod{37}$. Applying the process of successive squaring to -2, we have

   $$\begin{aligned}
   (-2)^2 &\equiv 4 \pmod{37} \\
   (-2)^4 &\equiv 4^2 \equiv 16 \pmod{37} \\
   (-2)^8 &\equiv 16^2 \equiv 256 \equiv 34 \equiv -3 \pmod{37}.
   \end{aligned}$$

   Finally, we have that

   $$109^{10} \equiv (-2)^2 \cdot (-2)^8 \equiv 4 \cdot -3 \equiv -12 \equiv \boxed{25} \pmod{37}.$$