Homework 22 Solutions

1. (a) **Compute $\phi(29)$. (Here $\phi(n)$ is Euler's phi function, as discussed in lecture.)**

   Recall that $\phi(n)$ is defined to be the number of numbers in the range $0, 1, 2, \ldots, n-1$ which are relatively prime to n. Since 29 is prime, all of these numbers except 0 are relatively prime to 29, and thus $\phi(29) = \boxed{28}$.

   (b) $\phi(77)$.

   Recall from lecture that if $n = pq$ where p and q are distinct primes, then $\phi(n) = (p-1)(q-1)$. Since $77 = 11 \cdot 7$, we see that $\phi(77) = (11-1)(7-1) = \boxed{60}$.

   (c) $\phi(8)$.

   In this case, we don't have a formula to use to compute ϕ (yet). Nonetheless, 8 is small enough that we can do it by hand. We wish to count how many of $0, 1, 2, 3, 4, 5, 6,$ and 7 are relatively prime to 8. It's easy to see that $1, 3, 5,$ and 7 are relatively prime to 8 and the others aren't. Thus $\phi(8) = \boxed{4}$.

2. (a) **Compute $12^{361} \pmod{77}$.**

   We see that 12 and 77 are relatively prime, and thus we can apply Euler's theorem. In particular, we know from part (a) that $\phi(77) = 60$, and thus Euler's theorem says that $12^{60} \equiv 1 \pmod{77}$. Hence we have

   $$12^{361} \equiv \left(12^{60}\right)^6 \cdot 12 \equiv \boxed{12} \pmod{77}.$$

   (b) **Compute $3^{402} \pmod{8}$.**

   Again, 3 and 8 are relatively prime, so we can use Euler's theorem. Because $\phi(8) = 4$, it says that $3^4 \equiv 1 \pmod{8}$. Thus

   $$3^{402} \equiv \left(3^4\right)^{100} \cdot 3^2 \equiv 9 \equiv \boxed{1} \pmod{8}.$$