

## Homework 29 Solutions

1. **Suppose, as discussed in today's lecture, that you wish to run the sieve of Eratosthenes on the numbers between 2 and 200.**

- (a) **After the first pass (that is, after keeping the 2 as prime and then crossing out all the multiples of 2) how many numbers are left? You don't need to actually run the sieve; instead you can use the counting methods we've discussed.**

We start with the numbers between 2 and 200, of which there are 199. The numbers we cross out in the first pass are 4, 6, 8, 10, ... 200. Dividing this sequence by 2, we see that it has the same number of numbers as the sequence 2, 3, 4, ... 100. Since this sequence has 99 numbers, we conclude that we cross out 99 numbers in the first pass. That means that the number which are left is  $199 - 99 = \boxed{100}$ .

- (b) **What percentage of the original numbers are left after this first pass?**

We know that 100 out of 199 are left. Since  $100/199 = 0.5025\dots$ , we see that approximately  $\boxed{50.25\%}$  are left.

- (c) **After the second pass (that is, after keeping the 3 as prime and then crossing out all the remaining multiples of 3) how many numbers are left? Again, you can do this by counting.**

In this second pass, we cross out 9, 15, 21, ... 195; that is, we start at 9 and then cross out every 6th number. This is because we've already crossed out the even numbers, so the remaining multiples of 3 are the odd multiples of 3. If we subtract 3 from this sequence, we get the sequence 6, 12, 18, ... 192. If we then divide this sequence by 6, we get 1, 2, 3, ... 32. Thus, we see that we cross out 32 numbers in the second pass. Since we have 100 numbers before this pass, afterwards we have  $100 - 32 = \boxed{68}$ .

- (d) **What percentage of the original numbers are left after this second pass?**

We know that 68 out of 199 are left. Since  $68/199 = 0.3417\dots$ , we see that approximately  $\boxed{34.17\%}$  are left.

2. (a) **Using the sieve of Eratosthenes, find all the primes between 160 and 200.**

Doing this, we find that the primes between 160 and 200 are

$$\boxed{163, 167, 173, 179, 181, 191, 193, 197, \text{ and } 199}.$$

- (b) **What percentage of the numbers between 160 and 200 are prime?**

Counting the number of primes we found in the last part, we see that there are 9 primes between 160 and 200. Also, there are 41 total numbers between 160 and 200. Since  $9/41 = 0.2195\dots$ , we see that approximately  $\boxed{21.95\%}$  of the numbers between 160 and 200 are prime.