Homework 3 Solutions

Problems

1. **How many fraternity names consisting of three Greek letters can be formed? (There are 24 letters in the Greek alphabet.) What if we do not allow repeats? What if we allow repeats, but we don't allow all three letters to be the same?**

   There are 24 choices for each letter, so by the multiplication principle, the total number of choices is $\boxed{24^3}$.

   If the letters must be distinct, there are 24 choices for the first letter, 23 for the second, and 22 for the third; the total number of choices is therefore $\boxed{24 \cdot 23 \cdot 22}$.

   If the only condition is that not all three letters be the same, we proceed using inclusion/exclusion. The number of excluded possibilities is 24 (one for each letter); we subtract this from the total number of choices to give $\boxed{24^3 - 24}$.

   Common Mistake: Many students answered the third part with $24 \cdot 24 \cdot 23$. This is incorrect because the number of choices for the third letter is 24 or 23 accordingly as the first two letters are different or the same. The multiplication principle does not apply because the number of choices for the third letter is not independent of previous choices.

2. **How many 4-digit numbers are divisible by 7? How many are divisible by 11? How many are divisible by neither 7 nor 11?**

   The 4-digit numbers are those between 1000 and 9999, inclusive. To count the ones divisible by 7, we must find the length of the list $1001, 1008, \ldots, 9996$. Dividing each number in this list by 7 gives a new list of the same length, $143, 144, \ldots, 1428$. Thus there are $1428 - 143 + 1 = \boxed{1286}$ 4-digit numbers divisible by 7.

   To count the 4-digit numbers divisible by 11, we must find the length of the list $1001, 1012, \ldots, 9999$. Dividing each number in this list by 11 yields the list $91, 92, \ldots, 909$ of the same length. But there are clearly $909 - 91 + 1 = \boxed{819}$ numbers in this new list.

   For the last part, we use the subtraction principle. The total number of 4-digit numbers is $9999 - 1000 + 1 = 9000$. From this we subtract those that are divisible by 7 (1286 such) and those that are divisible by 11 (819 such). But wait! We've subtracted some numbers twice, namely the ones that are divisible by both 7 and 11. These are just the multiples of 77: $1001, 1078, \ldots, 9933$. Dividing each number in this list by 77 gives the new list $13, 14, \ldots, 129$, which has length $129 - 13 + 1 = 117$. Since we subtracted these numbers twice, we must add them back into the total, to get $9000 - 1286 - 819 + 117 = \boxed{7012}$.

3. **Suppose the Harvard Texas Club consists of 100 students: 30 from the Houston area, 20 from Dallas/Fort Worth, 20 from San Antonio, and 30 from elsewhere. The Club wishes to elect officers (president, vice president, secretary, and treasurer). How many ways are there to do this? How many choices are there if all of the officers must be from the Houston area? What if at least one of the officers must be from Houston?**

   For this problem we will assume that a person cannot hold more than one office.

   For the first part, we are simply choosing a sequence of distinct people from a pool of 100, so by the multiplication principle there are $\boxed{100 \cdot 99 \cdot 98 \cdot 97}$ ways to elect the officers.

   The second part is similar, except the pool has shrunk to 30 people because thay must all be from Houston. So in this case there are $\boxed{30 \cdot 29 \cdot 28 \cdot 27}$ ways to elect the officers.

   For the third part we must use the subtraction principle. If at least one of the officers must be from Houston, we are excluding those selections in which all of the officers are from somewhere else besides Houston. There are $70 \cdot 69 \cdot 68 \cdot 67$ ways of electing officers, all of which hail from

outside of Houston (selecting a sequence of 4 distinct people from a pool of $100 - 30 = 70$), and subtracting this from the total as given in the first part of this problem yields the answer $\boxed{100 \cdot 99 \cdot 98 \cdot 97 - 70 \cdot 69 \cdot 68 \cdot 67}$.