

Homework 8 Solutions

1. Describe the numbers which are divisible by both 4 and 10.

From lecture (or the book), we know that the numbers divisible by two numbers, say a and b , are precisely the multiples of the least common multiple of a and b . So we need to find the least common multiple of 4 and 10. The first few multiples of 4 are 4, 8, 12, 16, 20, 24, 28, \dots . The first few multiples of 10 are 10, 20, 30, 40, \dots . Comparing these lists, we see that the least common multiple is 20, since it is the smallest number on both of these list. Thus, the numbers divisible by both 4 and 10 are precisely all the multiples of 20.

2. Determine the least common multiple of 12 and 15.

Again, we list the first few multiples of 12 and 15 and look for the smallest number to appear on both lists. The first few multiples of 12 are 12, 24, 36, 48, 60, 72, \dots . The first few multiples of 15 are 15, 30, 45, 60, 75, \dots . Comparing these, we see that the least common multiple of 12 and 15 is 60.

3. Determine the least common multiple of 5 and 7.

Once again, we list the first few multiples of 5 and 7 and look for the smallest number to appear on both lists. The first few multiples of 5 are 5, 10, 15, 20, 25, 30, 35, \dots . The first few multiples of 7 are 7, 14, 21, 28, 35, \dots . Comparing these, we see that the least common multiple of 12 and 15 is 35.

4. Describe the numbers which are divisible by 3, 5, and 6 (that is, the numbers which are divisible by all three of these).

This is slightly different from anything we've done so far. Nonetheless, we start by observing that the LCM of 3 and 5 is 15, and thus the number which are divisible by both 3 and 5 are precisely the multiples of 15. Said another way, the numbers divisible by 3 and 5 are precisely the numbers divisible by 15. Thus, the number we're looking for can also be described as the numbers divisible by 15 and 6. This is a problem we know how to do. As above, we can list the multiples of 15 and 6, and we see that their least common multiple is 30. We conclude that the numbers divisible by 3, 5, and 6 are precisely all the multiples of 30.