Homework 9 Solutions

1. **For the following pairs of numbers, compute the greatest common divisor:**

   (a) **18 and 32;**
   The Euclidean algorithm applied to 18 and 32 gives

   $$\begin{aligned} 32 &= 1 \cdot 18 + 14 \\ 18 &= 1 \cdot 14 + 4 \\ 14 &= 3 \cdot 4 + 2 \\ 4 &= 2 \cdot 2 + 0 \end{aligned}$$

   We conclude that the greatest common divisor of 18 and 32 is $\boxed{2}$.

   (b) **25 and 40;**
   The Euclidean algorithm applied to 25 and 40 gives

   $$\begin{aligned} 40 &= 1 \cdot 25 + 15 \\ 25 &= 1 \cdot 15 + 10 \\ 15 &= 1 \cdot 10 + 5 \\ 10 &= 2 \cdot 5 + 0 \end{aligned}$$

   We conclude that the greatest common divisor of 25 and 40 is $\boxed{5}$.

   (c) **27 and 36.**
   The Euclidean algorithm applied to 27 and 36 gives

   $$\begin{aligned} 36 &= 1 \cdot 27 + 9 \\ 27 &= 3 \cdot 9 + 0 \end{aligned}$$

   We conclude that the greatest common divisor of 27 and 36 is $\boxed{9}$.

2. **For each of the pairs in Problem 1, compute the least common multiple. What is the product of the greatest common divisor and the least common multiple? How does this compare to the product of the two numbers?**

   (a) **18 and 32;**
   From the first problem, we know that their gcd is 2. Hence, using the formula from lecture, $\operatorname{lcm}(a, b) = a \cdot b / \gcd(a, b)$, we see that $\operatorname{lcm}(18, 32) = \boxed{18 \cdot 32/2}$. This also implies that the product of the lcm and the gcd of 18 and 32 is just $18 \cdot 32$, the product of the numbers themselves.

   (b) **25 and 40;**
   From the first problem, we know that their gcd is 5. Hence, we see that $\operatorname{lcm}(25, 40) = \boxed{25 \cdot 40/5}$. Again, we see that the product of the lcm and the gcd of 25 and 40 is just $25 \cdot 40$, the product of the numbers themselves.

   (c) **27 and 36.**
   From the first problem, we know that their gcd is 9. Thus, $\operatorname{lcm}(27, 36) = \boxed{27 \cdot 36/9}$. The product of the lcm and the gcd of 27 and 36 is just $27 \cdot 36$, the product of the numbers themselves.

3. **Write down all of the common divisors of 120 and 96. How are these common divisors related to the greatest common divisor of these two numbers?**

   The divisors of 120 are 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, and 120. The divisors of 96 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, and 96. Looking at the lists, the divisors in common are $\boxed{1, 2, 3, 4, 6, 8, 12, \text{ and } 24}$, which are precisely the divisors of their gcd, 24.