Homework A Solutions

Problems

1. **What is the 17th Fibonacci number?**

   We list the Fibonacci numbers, starting with the 1st and 2nd, which are both 1:

   $$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, \ldots$$

   Here each number is obtained as the sum of the previous two. The 17th number in the list is $\boxed{1597}$.

2. **If the 20th Fibonacci number is 6765 and the 21st is 10946, what is the 23rd Fibonacci number?**

   The 22nd Fibonacci number is the sum of the 20th and the 21st, or $6765+10946 = 17711$. Then the 23rd Fibonacci number is the sum of the 21st and the 22nd, or $10946 + 17711 = \boxed{28657}$.

3. **For each of the first 10 Fibonacci numbers, determine whether it is odd or even. Do you see a pattern? Is the 15th Fibonacci number odd or even? The 100th? The 1,000th?**

   Consulting the above list, the pattern is:

   $$\text{odd, odd, even, odd, odd, even, odd, odd, even, odd}$$

   Clearly, every third Fibonacci number is even, and the others are odd. 15 is divisible by 3, so the 15th Fibonacci number is even. Neither 100 nor 1,000 is divisible by 3, so the 100th and 1,000th Fibonacci numbers are both odd.

Web Problem

1. **Which of the first 100 Fibonacci numbers are prime? (that is, not divisible by any positive number other than itself and 1). Do you notice a pattern? Do you think the 1,000th Fibonacci number could be prime?**

   From the Fibonacci Numbers website, the prime Fibonacci numbers among the first 100 are the 3rd, 4th, 5th, 7th, 11th, 13th, 17th, 23rd, 29th, 43rd, 47th, and 83rd. Notice that except for 4, these numbers are all prime. This pattern suggests that the 1,000th Fibonacci number is not prime, since 1,000 is not prime.

   In fact, if n is divisible by k, then the nth Fibonacci number is divisible by the kth Fibonacci number (but this fact is not obvious). Thus, as soon as n is bigger than 4, n must be prime for the nth Fibonacci number to have a chance at being prime. But the converse is not true: 19 is prime, but the 19th Fibonacci number is not prime.