

# Practice Questions for 1st Midterm

The following consists of practice questions to help you prepare for the midterm. They are similar in style and difficulty (although some of these may be a bit harder) to what will be on the midterm, although also more numerous (the midterm will consist of six questions, all of reasonable length). We won't be collecting or grading them, but we encourage you to work through them. We will be posting solutions sometime on Monday.

- How many numbers are there between 1000 and 5999?
  - Among the numbers of part (a) how many of them have none of their digits the same? (E.g., 4984 is not allowed but 4985 is OK.)
  - Among the numbers of part (a) how many of them have exactly two of their digits the same?
  - Among the numbers of part (a) how many of them are not divisible by 6?
  - Among the numbers of part (a) how many of them are not divisible by 6 or by 9?
- How many numbers are there between 11 and 111?
  - How many of them are even and not divisible by 3?
  - How many of them are odd and divisible by 3?
  - How many of them are divisible by neither 2, 3 or 37?
- Please compute the following (that is, your answer should be a whole number).
  - $\binom{5}{3}$
  - $\binom{1,000,113}{1,000,112}$
  - $\binom{6}{4,2,1,1}$
  - $\binom{4}{1,1,1,1}$
- Suppose that a license plate has seven spots, and that valid characters for these slots are the 26 letters (A through Z) and the 10 digits (0 through 9). How many possible license plates are there?
  - Now suppose that you want your license plate to contain exactly 3 letters. How many possible plates are there?
  - Now suppose that, in addition to having exactly 3 letters, you also refuse to allow these letters to be all in a row. How many possible plates are there meeting these conditions?
- How many two-card hands are there out of a deck of 52 that consists of two cards of contiguous denominations (that is a hand that looks like  $\{A, 2\}$ , or  $\{2, 3\}$ , or  $\{3, 4\}$ ,  $\dots$ , or  $\{K, A\}$ )?
  - If you are dealt two cards at random, what is the probability that you have such a two-card hand?
  - What is the probability of drawing a poker hand having exactly two pairs (recall that a pair is two cards of the same denomination), one of the pairs consisting of red suits (hearts, diamonds) and the other pair consisting of black suits (spades, clubs)?

6. (a) You are dealt, at random, 4 cards from a standard 52 card deck. What is the probability that the denominations are all consecutive (that is, you have the 4-card version of a straight)? Here the Ace can be either high or low, so that  $\{A, 2, 3, 4\}$  and  $\{J, Q, K, A\}$  both count.
- (b) What is the probability that you have 2 pair, that is, that you have two cards of one denomination and two cards of a different denomination?
7. (a) How many different (“mathematical”) words can be made by rearranging the letters of the word INSTINCTS?
- (b) By a palindrome we mean a (“mathematical”) word that is the same when spelled backwards, like RACECAR. How many different palindromes can be made by rearranging the letters of the word INSTINCTS?
8. (a) How many six-letter words are there with more consonants than vowels, but at least one vowel?
- (b) What if instead we want to count words with exactly five consonants but no repeated letters?
- (Assume that Y is counted as consonant.)
9. In the game of Yahtzee five dice are thrown simultaneously. What’s the probability...
- (a) ...to throw a pair and a triple (but not all five equal)?
- (b) ...to have exactly two pairs? (like 2, 2, 3, 3, 4 but NOT 4, 4, 3, 3, 3 or 5, 5, 5, 5, 6.)
- (c) ...that there are at least four equal?
10. Florian, Grigor and Rob have an old tradition of playing poker every Sunday afternoon. One day, desperate in his desire to buy a new Porsche, Rob decides to try to cheat and brings to the game a deck of cards identical to the one they are playing with. During one of the games he gets two aces. What is the probability that if he replaces two of his cards with aces from the remaining two suits (so that he has 4 aces) Florian and Grigor won’t be able to tell (by which we mean that they won’t have either of those aces in their own hands)?
11. (a) What is the greatest common divisor of 65 and 104?
- (b) What is their least common multiple?
- (c) How many numbers between 100 and 1000 are divisible by neither 4 nor 10?
12. A school goes on a trip to Smily River with 6 school buses. Once they reach the river, the kids go on a boat trip in 21 boats. A teacher notices that both in the buses and in the boats there was an equal number of kids and she knows that there are no more than 60 kids in the school. How many kids are there on the trip?
13. (a) Use the Euclidean algorithm to find the greatest common divisor of 15 and 63?
- (b) What is the least common multiple of 14 and 63?
- (c) Use the Euclidean algorithm to find the greatest common divisor of 14 and 65?

**Just in case you want even more questions to look at, we've provided a few extras. We won't be posting solutions for these.**

14. (a) How many numbers are there between 410 and 510, inclusive?  
(b) Of the numbers in (a), how many are divisible by 4?  
(c) Of the numbers in (a), how many are divisible by 4 but not by 10?
15. (a) How many "mathematical words" can be formed from the letters BARRYMAZUR?  
(b) In how many of these "words" does the letter R never appear twice in a row?
16. (a) Florian finds himself in a 3-story Walmart. Not only is each floor laid out in a perfect 2 by 4 grid, but there is an escalator at each intersection in the grid. If Florian starts at the north-east corner of the top floor and the only exit is at the south-west corner of the bottom floor, his quickest ways out are by going west four aisles, south two aisles, and down two escalators. How many such options does he have for leaving the building?  
(b) Suppose that Florian is afraid of escalators and, as a result, refuses to take two back-to-back trips on the escalators. How many ways can he leave now?  
(c) Just to make sure that you remember the formulas for computing with some of the notation we've defined, please compute the actual numbers corresponding to your answers on the last two parts.
17. The game of Euchre is played by four players with a deck of 24 cards, each player being dealt five (leaving four cards undealt).  
(a) What is the number of possible deals?  
(b) Explain why the number in (a) must be a multiple of 24.
18. Prof. Elkies gives a piano recital and he needs to choose several pieces to play. He can choose between 5 Beethoven piano sonatas, 10 Chopin piano pieces and 7 of his own compositions. Assume that he does not repeat a piece during a recital or practice.  
(a) In how many ways can he choose a program of 3 pieces if he wants to play a Beethoven piece first?  
(b) The organizers of the recital require a sequence of 3 pieces and that each composer is represented at least once. How many possible programs are there now?  
(c) During practice for a different recital, Prof. Elkies plays two of the pieces. He is known to sometimes whistle or hum in accompaniment to his own pieces, but not both. If you take this into account, how many possibilities can he choose from? (Here we count a piece with whistling as different from the same piece with humming or with no accompaniment.)
19. (a) Suppose that the Harvard math department is headed by a chairman, a vice-chairman, and a 3-member advisory committee. If the department has 19 faculty members eligible for these positions, how many ways are there to choose the department leadership? (Note that no one can hold more than one position, e.g. the chairman cannot also serve on the advisory committee.)  
(b) Now suppose that the 19 faculty consist of 13 full professors and 6 assistant professors. If the chairman must be a senior professor and the advisory committee must have at least 1 senior professor, how many ways can the leadership be chosen?
20. Explain why  $\binom{52}{5}$  is equal to  $\binom{51}{5} + \binom{51}{4}$ .