Practice Questions for 2nd Midterm

The following consists of practice questions to help you prepare for the midterm. They are similar in style and difficulty (although some of these may be a bit harder) to what will be on the midterm, although also more numerous (the midterm will consist of six questions, all of reasonable length). We won't be collecting or grading them, but we encourage you to work through them. We will be posting solutions sometime on Monday.

1. (a) Use the Euclidean Algorithm to find the greatest common divisor of 44 and 17.
   (b) Find whole numbers x and y so that $44x + 17y = 1$ with $x > 10$.
   (c) Find whole numbers x and y so that $44x + 17y = 1$ with $y > 10$.

2. For each of the following four parts say whether there are whole numbers x and y satisfying the equation. If an equation has a solution, write down a possible choice of x and y.
   (a) $69x + 123y = 2$.
   (b) $47x + 21y = 2$.
   (c) $47x - 21y = 6$.
   (d) $49x + 21y = 6$.

3. (a) Is the binomial coefficient $\binom{12}{4}$ divisible by 11?
   (b) How many divisors does $\binom{12}{4}$ have?
   (c) How many of them are divisible by 3?

4. Let $m = 1100$ and $n = 2^2 \times 3^3 \times 5^5$.
   (a) Compute $\gcd(m, n)$.
   (b) How many whole numbers divide m but not n?
   (c) How many whole numbers divide n but not m?

5. Do the following calculations.
   (a) $7 \cdot 9 \pmod{36}$.
   (b) $8 - 21 \pmod{31}$.
   (c) $68 \cdot 69 \cdot 71 \pmod{72}$.
   (d) $108! \pmod{83}$.
   (e) $60^{59} \pmod{61}$.
   (f) $1/2 \pmod{17}$.
   (g) $1/11 \pmod{43}$.
   (h) $1/2 \pmod{8}$.

6. (a) What is the last digit of 3^{10}?
   (b) Compute $2^{(3^{10})} \pmod{11}$. (Note that this is *not* the same as $(2^3)^{10} \pmod{11}$.)

(c) Compute $3^{(2^{10})}$ (mod 11).

7. (a) Find an x between 0 and 19 such that $x^2 \equiv 5 \bmod 19$.
   (b) What does Fermat's theorem say about powers of x?
   (c) Compute $5^9 \bmod 19$.

8. (a) Use the Euclidean Algorithm to find the reciprocal of 40 mod 93. Check your work by verifying that your answer is in fact a solution of $40x \equiv 1 \bmod 93$.
   (b) Using your answer to the first part, find the reciprocals mod 93 of 4 and 89. (Hint: $4 + 89 = 93$.)

9. (a) Which of the numbers 90, 91, 92, ..., 100 has a reciprocal mod 100?
   (b) Choose two of the numbers you found in the first part and compute their reciprocals mod 100.

10. The goal of this problem is to find reciprocals mod 21 for all the numbers mod 21 that have such a reciprocal. Record your answers in the table below.

x	0	1	2	3	4	5	6	7	8	9
$1/x$	NONE	1								

x	10	11	12	13	14	15	16	17	18	19	20
$1/x$											

   (a) Identify all the numbers x other than 0 that have no reciprocal mod 21, and enter NONE in the $1/x$ box of every such number.
   (b) What is $\frac{1}{20}$ mod 21?
   (c) Use the fact that $2^6 \equiv 64 \equiv 1 \bmod 21$ to find the reciprocals of 2, 4, 8, and 16.
   (d) fill in the rest of the table.

11. Please make the requested computations modulo 11 putting your answers in the range

$$\{0, 1, 2, \ldots, 10\}.$$

   (a) Find 3^{12} modulo 11.
   (b) Find $2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \pmod{11}$.
   (c) Does a solution to the equation

$$5^{10} y \equiv 6^{61} \pmod{11}$$

   exist? If it does, please find it.

12. Prof. Mazur goes to the supermarket and buys several dozen eggs. He uses them to make several batches of his famous crème brûlée. Each batch requires 7 eggs. When he's done cooking, he notices that he has 4 eggs left over. If he knows he bought less than 10 dozen eggs, how many dozen did he buy?

13. Florian is running laps on a small track. In fact, it takes him exactly 17 seconds to run a lap. After running for a while, he has run a whole number of laps and he notices that the second hand on his watch has advanced 6 seconds. If he knows he ran less than 70 laps, how many laps did he run?

14. (a) What is the 3rd root of $9 \pmod{29}$?

    (b) What is the 37th root of $6 \pmod{41}$?

    (c) Find all square roots of $2 \pmod{7}$.