

## Additional Practice Problems

- How many numbers are there between 312 and 3005 (inclusive) which are divisible by 3 but not by 6?
  - How many numbers in the previous problem are not divisible by 3?
- How many numbers are there between 160 and 1050, inclusive?
  - Of the numbers in (a), how many are divisible by 3? How many are divisible by 4?
  - Of the numbers in (a), how many are divisible by 3 but not by 4?
  - Of the numbers in (a), how many are divisible by 3 *or* by 4 (or both)? (Examples of such numbers are 198, 200, 201.)
- The English alphabet has 26 letters, 6 of which (A, E, I, O, U, Y) are vowels.
  - How many possible fifteen-letter English words are there? (“Mathematical words”, that is: as usual we do not ask that the words make any sense — it’s all English to us in any case.)
  - Of the fifteen-letter English words, how many contain exactly four vowels?
  - A *palindrome* is a word, such as SEES or DEIFIED or EEEEEEEEE, that reads the same forward and back. Of the fifteen-letter English words with exactly four vowels, how many are palindromes?
- Suppose that a license plate has eight spots, and that valid characters for these slots are the 26 letters (A through Z) and the 10 digits (0 through 9). How many possible license plates are there?
  - Now suppose that you want your license plate to contain exactly 4 letters. How many possible plates are there?
  - Now suppose that, in addition to having exactly 4 letters, you also refuse to allow these letters to be all in a row. How many possible plates are there meeting these conditions?
- You are dealt, at random, 6 cards from a standard 52 card deck. What is the probability that the denominations are all consecutive (that is, you have the 6-card version of a straight)? Here the Ace can be either high or low, so that  $\{A, 2, 3, 4, 5, 6\}$  and  $\{9, 10, J, Q, K, A\}$  both count.
  - What is the probability that you have 3 pair, that is, that you have two cards of one denomination and two cards of a different denomination and two cards of a third (yet different) denomination?
- You are playing a slightly unorthodox version of Scrabble where you are given seventeen letters (rather than the usual seven). Amazingly enough, you drew (randomly) from the pile precisely these seventeen letters: TRISKAIDEKAPHOBIA, but probably not in exactly that order. Given that you drew this set of seventeen Scrabble tiles, what is the probability that you did, in fact, draw them from the pile spelling the word TRISKAIDEKAPHOBIA?
- Yahtzee is played with 5 dice, each numbered in the usual way from 1 to 6 and equally likely to land on each face.
  - What is the probability of the total on the 5 dice being at least 28?

- (b) Suppose now that one of the dice is marked dishonestly, with the 1-face replaced by a second 6 (so the faces are 2, 3, 4, 5, 6, 6 instead of the honest 1, 2, 3, 4, 5, 6). What is the probability of scoring at least 28 with this die and four honest dice?
8. At parties, local mathematician George Nomial often produces an ordinary pack of cards and invites a stranger to choose three at random. George will then claim that at least one of the chosen cards is an Ace or a King. The thing is: there's no trick involved! What's the probability that George's claim is correct (i.e., that at least one of the three is an Ace or King)?
9. (a) Compute  $\binom{16}{4}$  as a whole number.  
(b) Compute  $\binom{17}{13}$  as a whole number.  
(c) Yes or no, and why: Is  $\binom{107}{104}$  bigger than  $\binom{106}{4} + \binom{106}{3}$ ?
10. You go shopping at the Pencil Store, and need to buy 100 pens. There are green pencils and purple pencils. In how many ways can you choose 100 of them?