

Quantitative Reasoning 28: The Magic of Numbers

Homework 27

Assigned on Monday December 10th
Due at 12 noon **Wednesday** December 12th

Please submit problem sets at the end of the relevant lecture, or leave in the box labeled QR28 outside the Math Department's main office, on the third floor of the Science Center (Room 325).

Reading:

Gross-Harris, Chapter 23

Problems:

Please explain your reasoning and show your work.

1. A number b between 1 and $n - 1$ is called a *witness* for the fact that n is composite if $b^{n-1} \not\equiv 1 \pmod{n}$. (remember: if n were prime, Fermat's little theorem would say that $b^{n-1} \equiv 1 \pmod{n}$. So n has to be composite if it has any witness).
 - (a) Show that 3 fails to be a witness for 91 to be composite.
 - (b) Show that 2 is a witness for 255 to be composite.
2. Find two witnesses to the fact that 121 is composite.