

Quantitative Reasoning 28: The Magic of Numbers

Homework 20

Assigned on Friday November 9th

Due at 12 noon Wednesday November 14th

Please submit problem sets at the end of the relevant lecture, or leave in the box labeled QR28 outside the Math Department's main office, on the third floor of the Science Center (Room 325).

Reading:

Gross-Harris, Chapter 18

Problems:

Please explain your reasoning and show your work.

1. Use repeated squaring (a.k.a., the “doubling method”) to find the following powers in modular arithmetic:
 - (a) $2^{41} \pmod{63}$ (Hint: $41 = 32 + 8 + 1$).
 - (b) $(-3)^{100} \pmod{78}$ (Hint: $100 = 64 + ? + ?$).
 - (c) $70^{49} \pmod{67}$.
2.
 - (a) What does Fermat's Theorem say about powers $\pmod{53}$?
 - (b) Compute $3^{109} \pmod{53}$.
 - (c) Compute $2^{270} \pmod{53}$.
3.
 - (a) What is the last digit of 3^{991} ?
 - (b) Compute $3^{991} \pmod{11}$.
 - (c) Compute $26^{991} \pmod{13}$.
4.
 - (a) Create a power table for arithmetic $\pmod{13}$. This will be a table whose rows correspond to numbers in arithmetic $\pmod{13}$ (that is,

the numbers $\{0, 1, 2, \dots, 12\}$), and whose entries are their various powers. Compute the powers from the 1st up to the 13th power for each number. (Remember, for example, that $10 = -3 \pmod{13}$ and you can use this to avoid doing the computations for 10 once you've done them for 3.)

(b) Compute $2^{742} \pmod{13}$.