Practice Questions for 2nd Midterm

Please do not get discouraged; this is longer and harder than the real midterm will be.

1. (a) Use the Euclidean Algorithm to find the greatest common divisor of 44 and 17.

   (b) Find whole numbers x and y so that $44x + 17y = 1$ with $x > 10$.

   (c) Find whole numbers x and y so that $44x + 17y = 1$ with $y > 10$.

2. For each of the following four parts say whether there are whole numbers x and y satisfying the equation. If an equation has a solution, write down a possible choice of x and y.

   (a) $69x + 123y = 2$.

   (b) $47x + 21y = 2$.

   (c) $47x - 21y = 6$.

   (d) $49x + 21y = 6$.

3. (a) What is the largest prime number dividing the binomial coefficient $\binom{12}{4}$?

   (b) How many divisors does $\binom{12}{4}$ have?

   (c) How many of the divisors of $\binom{12}{4}$ are divisible by 3?

4. Let $m = 1100$ and $n = 2^2 \times 3^3 \times 5^5$.

   (a) Compute $\gcd(m, n)$.

   (b) Compute $\mathrm{lcm}(m, n)$.

   (c) How many whole numbers divide m but not n?

   (d) How many whole numbers divide n but not m?

5. Do the following calculations. As always, when working mod n, leave your answer in the range $0, 1, \ldots, n-1$.

   (a) $7 \cdot 9 \pmod{36}$.

   (b) $8 - 21 \pmod{31}$.

   (c) $68 \cdot 69 \cdot 71 \pmod{72}$.

   (d) $108! \pmod{83}$.

   (e) $60^{59} \pmod{61}$.

   (f) $1/2 \pmod{17}$.

   (g) $1/11 \pmod{43}$.

6. (a) Compute $21^{4600} \pmod{47}$.

   (b) Compute $21^{4601} \pmod{47}$.

   (c) Compute $21^{4599} \pmod{47}$. (Hint: your work on 2(b) will help).

7. (a) Compute $87^{51} \pmod{47}$.

(b) Compute 94^{46} (mod 47).

8. (a) Find an x between 0 and 19 such that $x^2 \equiv 5$ mod 19.

   (b) What does Fermat's theorem say about powers of x?

   (c) Compute 5^9 mod 19.

9. (a) Use the Euclidean Algorithm to find the reciprocal of 40 mod 93. Check your work by verifying that your answer is in fact a solution of $40x \equiv 1$ mod 93.

   (b) Using your answer to the first part, find the reciprocals mod 93 of 4 and 89. (Hint: $4 + 89 = 93$.)

10. The goal of this problem is to find reciprocals mod 23 for all the nonzero numbers mod 23. Record your answers in the table below.

x	1	2	3	4	5	6	7	8	9	10	11
$1/x$	1										

x	12	13	14	15	16	17	18	19	20	21	22
$1/x$											

   (a) What is $\frac{1}{22}$ mod 23?

   (b) Use the fact that $2^{11} \equiv 2048 \equiv 1$ mod 23 to find the reciprocals of 2, 4, 8, and 16.

   (c) Fill in the rest of the table.

11. Please make the requested computations modulo 11 putting your answers in the range

$$\{0, 1, 2, \ldots, 10\}.$$

   (a) Find 3^{12} modulo 11.

   (b) Find $2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9$ (mod 11).

   (c) Does a solution to the equation

$$5^{10}y \equiv 6^{61} \pmod{11}$$

   exist? If it does, please find it.

12. Bob buys several dozen eggs. He uses them to make several batches of crème brûlée. Each batch requires 7 eggs. When he's done cooking, he notices that he has 4 eggs left over. If he knows he bought fewer than 10 dozen eggs, how many dozen did he buy?

13. William is running laps on a small track. In fact, it takes him exactly 17 seconds to run a lap. After running for a while, he has run a whole number of laps and he notices that the second hand on his watch has advanced 6 seconds. If he knows he ran fewer than 70 laps, how many laps did he run?