Homework 10 Solutions

Problems

1. **Use the Euclidean algorithm to find the greatest common divisor of 240 and 198.**

$$240 = 198 + 42$$

$$198 = 4 \times 42 + 30$$

$$42 = 30 + 12$$

$$30 = 2 \times 12 + 6$$

$$12 = 2 \times \boxed{6}$$

Therefore gcd(240, 198) = 6.

2. **Use the Euclidean algorithm to find the greatest common divisor of 2400 and 1987.**

$$2400 = 1987 + 413$$

$$1987 = 4 \times 413 + 335$$

$$413 = 1 \times 335 + 78$$

$$335 = 4 \times 78 + 23$$

$$78 = 3 \times 23 + 9$$

$$23 = 2 \times 9 + 5$$

$$9 = 1 \times 5 + 4$$

$$5 = 1 \times 4 + 1$$

$$4 = 4 \times \boxed{1}$$

Therefore gcd(2400, 1987) = 1.

3. (a) **Use the Euclidean algorithm to find the greatest common divisor of 11 and 29.**

$$29 = 2 \times 11 + 7$$
$$11 = 1 \times 7 + 4$$
$$7 = 1 \times 4 + 3$$
$$4 = 1 \times 3 + 1$$
$$3 = 3 \times \boxed{1}$$

Therefore gcd(11,29)=1.

(b) **Express 1 as a combination of 11 and 29.**

$$1 = 4 - 3$$
$$= 4 - (7 - 4) = 2 \times 4 - 7$$
$$= 2 \times (11 - 7) - 7 = 2 \times 11 - 3 \times 7$$
$$= 2 \times 11 - 3 \times (29 - 2 \times 11) = \boxed{8 \times 11 - 3 \times 29}$$

(c) **Express 2 as a combination of 11 and 29.**

$$2 = 2 \times (8 \times 11 - 3 \times 29)$$
$$= \boxed{16 \times 11 - 6 \times 29}$$

4. (a) **Use the Euclidean algorithm to find the greatest common divisor of 69 and 87.**

$$87 = 1 \times 69 + 18$$
$$69 = 3 \times 18 + 15$$
$$18 = 1 \times 15 + 3$$
$$15 = 5 \times \boxed{3}$$

Therefore gcd(69, 87) = 3.

(b) **Express 3 as a combination of 69 and 87.**

$$3 = 18 - 15$$
$$= 18 - (69 - 3 \times 18) = 4 \times 18 - 69$$
$$= 4 \times (87 - 69) - 69 = \boxed{4 \times 87 - 5 \times 69}$$

(c) **Can 1 be expressed as a combination of 69 and 87? Why or why not?**

$\boxed{\text{No,}}$ because any whole number that is a combination of 69 and 87 is a multiple of their GCD, 3.