

Homework 11 Solutions

Problems

- (a) Find the greatest common divisor of 161 and 205.
(b) Find whole numbers x and y so that $161x + 205y = 1$.

$$205 = 161 + 44$$

$$161 = 3 \times 44 + 29$$

$$44 = 29 + 15$$

$$29 = 15 + 14$$

$$15 = 14 + 1$$

Thus $\gcd(161, 205) = 1$

Running the process in reverse...

$$1 = 15 - 14$$

$$1 = 15 - (29 - 15) = 2 \times 15 - 29$$

$$1 = 2 \times (44 - 29) - 29 = 2 \times 44 - 3 \times 29$$

$$1 = 2 \times 44 - 3 \times (161 - 3 \times 44) = -3 \times 161 + 11 \times 44$$

$$1 = -3 \times 161 + 11 \times (205 - 161) = 11 \times 205 - 14 \times 161$$

And we are done.

- (a) Find the greatest common divisor of 111 and 234.
(b) Find whole numbers x and y so that $111x + 234y = 6$.

$$234 = 2 \times 111 + 12$$

$$111 = 9 \times 12 + 3$$

$$12 = 4 \times 3$$

So we see $\gcd(234, 111) = 3$.

Running the process in reverse we get

$$3 = 111 - 9 \times 12$$

$$3 = 111 - 9 \times (234 - 2 \times 111) = 19 \times 111 - 9 \times 234$$

So we see that $6 = 38 \times 111 - 18 \times 234$.

3. (a) Find the greatest common divisor of 186 and 270.
(b) For which whole numbers m is it possible to find whole numbers x and y so that $186x + 270y = m$?

$$270 = 186 + 84$$

$$186 = 2 \times 84 + 18$$

$$84 = 4 \times 18 + 12$$

$$18 = 12 + 6$$

$$12 = 2 \times 6$$

Hence $\gcd(270, 186) = 6$.

We can find x and y to solve the given equation whenever m is a multiple of 6.