

# Homework 12 Solutions

## Problems

1. **Describe how you could measure 2 cups of water given that you have three measuring jugs: one for 12 cups, one for 20 cups, and one for 30 cups.**

There are several ways of doing this. For example:  $2 = 20 + 30 - 4 \times 12$ , so we might fill the 30 cup jug, pour off 12 cups twice (by pouring into the 12 cup jug until full, and then dumping this on the ground). We could then fill the 20 cup jug, and pour off 12. We would now have 6 cups in one and 8 cups in the other. We could combine this into the 20 cup jug and again pour off 12 (the fourth 12), leaving  $6 + 8 - 12 = 2$  cups, as desired.

You might consider a more systematic way of doing this with an arbitrary combination of 20, 30 and 12. (Is this always possible?)

2. **Using the Sieve of Eratosthenes to compute prime numbers, find which of the numbers between 400 and 440 are prime.**

The square root of 440 is less than 21 since  $21^2 = 441$ . So the highest prime number we need to check is 19.

Kill off numbers divisible by 2:

401, 403, 405, 407, 409, 411, 413, 415, 417, 419, 421, 423, 425, 427, 429, 431, 433, 435, 437, 439

Kill off numbers divisible by 3:

401, 403, 407, 409, 413, 415, 419, 421, 425, 427, 431, 433, 437, 439

Kill off numbers divisible by 5:

401, 403, 407, 409, 413, 419, 421, 427, 431, 433, 437, 439

Kill off numbers divisible by 7:

401, 403, 407, 409, 419, 421, 431, 433, 437, 439

Kill off numbers divisible by 11:

401, 403, 409, 419, 421, 431, 433, 437, 439

Kill off numbers divisible by 13:

401, 409, 419, 421, 431, 433, 437, 439

Kill off numbers divisible by 17:

401, 409, 419, 421, 431, 433, 437, 439

Kill off numbers divisible by 19:

401, 409, 419, 421, 431, 433, 439

3. **Explain why none of the nine consecutive numbers  $10! + 2, 10! + 3, \dots, 10! + 10$  can be a prime number. (Hint: each of the numbers has a small factor.) This is called a “prime desert” of length 9.**

$10!$  is divisible by all the numbers between 2 and 10. So  $10! + 2$  is divisible by 2, and  $10! + 3$  is divisible by 3, and so on...

4. **Can you find a prime desert of [at least] length 99?**

Yes:  $100! + 2, 100! + 3, 100! + 4, \dots, 100! + 100$ .

Each term  $100! + i$  is divisible by  $i$ , hence not prime.