Homework 13 Solutions

Problems

1. **Factor the following into prime numbers:**

   (a) $\binom{20}{6}$**; (*Hint:* You do not need to compute the actual number.)**

   (b) 6006;

   (c) 2006.

$$\begin{aligned}\binom{20}{6} &= 20 \times 19 \times 18 \times 17 \times 16 \times 15 \times /6 \times 5 \times 4 \times 3 \times 2 \\ &= 19 \times 17 \times 8 \times 15 \\ &= 2^3 \times 3 \times 5 \times 17 \times 19\end{aligned}$$

$$\begin{aligned}6006 &= 6 \times 1001 \\ &= 2 \times 3 \times 7 \times 11 \times 13\end{aligned}$$

$$\begin{aligned}2006 &= 2 \times 1003 \\ &= 2 \times 17 \times 59\end{aligned}$$

2. **(a) Is** $\binom{19}{7}$ **divisible by 13?**

   (b) Is $\binom{19}{7}$ **divisible by 19?**

   (c) Is $\binom{19}{7}$ **divisible by 23?**

$$\binom{19}{7} = \frac{19 \times 18 \times 17 \times 16 \times 15 \times 14 \times 13}{7 \times 6 \times 5 \times 4 \times 3 \times 2}$$

First note that 13, 19, and 23 are all prime.

13 divides the top of the fraction, but not the bottom, hence 13 does divide $\binom{19}{7}$.

19 divides the top of the fraction, but not the bottom, hence 19 does divide $\binom{19}{7}$.

23 does not divide the top of the fraction (or the bottom), hence 23 does not divide $\binom{19}{7}$.