

Homework 14 Solutions

Problems

- (a) Is $\binom{19}{7}$ divisible by 10?
(b) Is $\binom{19}{7}$ divisible by 6?

$$\binom{19}{7} = \frac{19 \times 18 \times 17 \times 16 \times 15 \times 14 \times 13}{7 \times 6 \times 5 \times 4 \times 3 \times 2}$$

5 divides the top once and the bottom once, hence 5 (and so 10) does not divide $\binom{19}{7}$.

2 divides the top 6 times, and the bottom 4 times, hence 2 divides $\binom{19}{7}$. 3 divides the top 3 times, and the bottom twice, hence 3 divides $\binom{19}{7}$. So both 2 and 3 divide $\binom{19}{7}$ so 6 divides $\binom{19}{7}$.

- Find the gcd of 18000 and 10935 by factoring these numbers.

$$18000 = 2^4 \times 3^2 \times 5^3$$

$$10935 = 3^7 \times 5$$

Hence $\gcd(18000, 10935) = 3^2 \times 5 = 45$.

- Let $a = 2^3 \times 3^4 \times 11^2$ and $b = 2^2 \times 3^2 \times 5$.

(a) Compute $\gcd(a, b)$.

(b) Compute $\text{lcm}(a, b)$.

$$\gcd(a, b) = 2^{\min(3,2)} \times 3^{\min(4,2)} \times 5^{\min(0,1)} \times 11^{\min(2,0)} = 2^2 \times 3^2.$$

$$\text{lcm}(a, b) = 2^{\max(3,2)} \times 3^{\max(4,2)} \times 5^{\max(0,1)} \times 11^{\max(2,0)} = 2^3 \times 3^4 \times 5 \times 11^2.$$