Homework 17 Solutions

Problems

1. **Do the following computations in the given modulus.**

    (a) $6 - 4 \pmod 7$.

    (b) $80 + 21 \pmod{101}$.

    (c) $3 - 12 \pmod{15}$.

    (d) $456 \times 450 \pmod{457}$.

    $\boxed{2}$, $\boxed{0}$, $\boxed{6}$, $-1 \times -7 \equiv \boxed{7}$

2. **Do the following computations in the given modulus.**

    (a) $457 \times 458 \pmod{459}$.

    (b) $13 \times 44 \pmod 5$.

    (c) $13 \times 44 \pmod{15}$.

    In all three cases, we have $(-2) \times (-1) \equiv \boxed{2}$ in the given modulus.

3. **Use the Euclidean algorithm to compute the following.**

    (a) $1/13 \pmod{97}$.

    (b) $1/73 \pmod{17}$.

    We must solve for x in $1 = 13x + 97y$. Using the Euclidean algorithm, we find

    $$\begin{aligned} 97 &= 7 \times 13 + 6 \\ 13 &= 2 \times 6 + 1 \end{aligned}$$

    and running it backwards,

    $$\begin{aligned} 1 &= 13 - 2 \times 6 \\ &= 13 - 2 \times (97 - 7 \times 13) = 15 \times 13 - 2 \times 97 \end{aligned}$$

    so $1/13 \equiv \boxed{15} \pmod{97}$.

    Note that $73 \equiv 5 \pmod{17}$. Hence, we solve for x in $1 = 5x + 17y$. Using the Euclidean algorithm, we find

    $$\begin{aligned} 17 &= 3 \times 5 + 2 \\ 5 &= 2 \times 2 + 1 \end{aligned}$$

    and so

    $$\begin{aligned} 1 &= 5 - 2 \times 2 \\ &= 5 - 2 \times (17 - 3 \times 5) = 7 \times 5 - 2 \times 17 \end{aligned}$$

    so $1/73 \equiv \boxed{7} \pmod{17}$.