Homework 19 Solutions

Problems

1. **Find all the reciprocals of non-zero values mod 17.**

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$1/x$	1	9	6	13	7	3	5	15	2	12	14	10	4	11	8	16

First note that $16 \equiv -1$ so $16^{-1} \equiv 16$.

Then note that $1 \equiv 18 \equiv 35 \equiv 52 \pmod{17}$.

$9 \times 2 \equiv 18 \equiv 1 \pmod{17}$ so 9 and 2 are inverses.

$3 \times 6 \equiv 18 \equiv 1 \pmod{17}$ so 3 and 6 are inverses.

$7 \times 5 \equiv 35 \equiv 1 \pmod{17}$ so 7 and 5 are inverses.

$4 \times 13 \equiv 52 \equiv 1 \pmod{17}$ so 4 and 13 are inverses.

Also, $(-2)^{-1} = -2^{-1}$ so $-2 \equiv 15$ and $-9 \equiv 8$ are inverses.

Similarly, $(-3)^{-1} = -3^{-1}$ so $-3 \equiv 14$ and $-6 \equiv 11$ are inverses.

The remaining pair, 10 and 12 must be inverses (though we can check: $10 \times 12 \equiv 120 \equiv 1 \pmod{17}$.

We could have also done some of these using $(ab)^{-1} \equiv a^{-1}b^{-1}$ or using the Euclidean algorithm.

2. **Compute the following powers mod 13.**

   (a) $2^0, 2^1, 2^2, ..., 2^{15}, 2^{16}$.

   (b) $3^0, 3^1, 3^2, ..., 3^{15}, 3^{16}$.

First note that $2^0 \equiv 1$, $2^1 \equiv 2$, $2^2 \equiv 4$, $2^3 \equiv 8$, and $2^4 \equiv 16 \equiv 3 \pmod{13}$.

Then $2^5 \equiv 2 \times 3 \equiv 6$ and $2^6 \equiv 2 \times 6 \equiv 12 \equiv -1$, which means that the next 6 values will be negatives of the first:

$2^7 \equiv -2 \equiv 11$, $2^8 \equiv -4 \equiv 9$, $2^9 \equiv -8 \equiv 5$, $2^{10} \equiv -3 \equiv 10$, $2^{11} \equiv -6 \equiv 7$, and $2^{12} \equiv -(-1) \equiv 1$.

Thereafter they repeat, i.e., $2^{13} \equiv 2^1 \equiv 2$, $2^{14} \equiv 2^2 \equiv 4$, $2^{15} \equiv 2^3 \equiv 8$, and $2^{16} \equiv 2^4 \equiv 3$.

$3^0 \equiv 1$, $3^1 \equiv 3$, $3^2 \equiv 9$, $3^3 \equiv 27 \equiv 1$, and they repeat from here.