

# Homework 23 Solutions

## Problems

1. **Compute the following:**

(a)  $\phi(29)$ . (**Here  $\phi(n)$  is Euler's phi function, as discussed in lecture.**)

(b)  $\phi(77)$ .

(c)  $\phi(6615)$ .

$$\phi(29) = \boxed{28} \text{ since } 29 \text{ is prime.}$$

$$\phi(77) = (6/7)(10/11)77 = \boxed{60}$$

$$6615 = 3^3 \times 5 \times 7^2 \text{ so } \phi(6615) = (2/3)(4/5)(6/7)6615 = \boxed{3024}$$

2.  $3^{28} = 1 \pmod{29}$  by **Fermat's Little Theorem** (since **29 is prime**).

**Compute**  $3^{60} \pmod{77}$ .

At the moment we can only do this by doubling. We discover that the answer is  $3^{60} = \boxed{1} \pmod{77}$  which hints at a generalization of Fermat's little theorem.