

Homework 25 Solutions

The first step in doing this problem set was to decode it. By comparing the portions of this problem set with the header, which doesn't change from one assignment to the next, we obtain the following letter correspondences:

Coded	A	B	C	D	E	F	G	H	I	J	K	L	M
Actual	G	I	A	E	S	H	D	X	W	Y	Z	C	U
Coded	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Actual	K	F	V	N	O	R	Q	T	B	P	M	L	J

We can therefore decode the single problem to get ...

1. Compute the seventh root of four (modulo eighty-five).

First we check that 4 and 85 are relatively prime, which they are. Now since $85 = 5 \cdot 17$, we can compute $\phi(85) = 4 \cdot 16 = 64$. Thus we want to solve $7x + 64y = 1$. The Euclidean Algorithm gives

$$64 = 9 \cdot 7 + 1,$$

and running it backwards,

$$1 = 64 - 9 \cdot 7.$$

We conclude that $4^{1/7} \equiv 4^{-9} \equiv 4^{55} \pmod{85}$, where we've again used Euler's theorem to make the exponent positive.

Finally, we need to compute $4^{55} \pmod{85}$. Successive squaring gives

$$4^2 \equiv 16 \pmod{85}$$

$$4^4 \equiv 256 \equiv 1 \pmod{85}.$$

Thus we have

$$4^{55} \equiv (4^4)^{13} \cdot 4^2 \cdot 4 \equiv 16 \cdot 4 \equiv \boxed{64} \pmod{85}.$$