

Homework 3 Solutions

Problems

1. **The American Students' Phonetic Association is trying to increase membership by pretending to be a fraternity. As part of this subterfuge it wants to pick a three-letter fraternity name but with names drawn from the phonetic alphabet which has 107 letters. How many names can be formed?**

How many names can be formed if each of the three letters has to be different?

What if we only disallow names where all three letters are the same (like $e e e$) but all other combinations are okay - how many possible names are there then?

For the first part we use the multiplication principle $\boxed{107^3}$.

For the second part we use the multiplication principle without repetition $\boxed{107 \cdot 106 \cdot 105}$.

The final part is most easily tackled using the subtraction principle. There are 107 possible names with all three letters being the same, hence the number of allowable names is

$$\boxed{107^3 - 107}.$$

2. **How many three-digit numbers are there?**

How many three-digit numbers are divisible by 3? How many three-digit numbers are divisible by 5?

How many three-digit numbers are neither divisible by 3 nor by 5? (*Hint: A number is divisible by both 3 and 5 exactly when it is divisible by 15.*)

We can tackle the first part by counting - the number of numbers between 100 and 999 is $\boxed{999 - 100 + 1 = 900}$.

We do the second part by counting as well. The number of numbers divisible by 3 between 100 and 999 is the number of numbers divisible by 3 between 102 and 999. This (dividing by 3) is the same as the number of numbers between 34 and 333 which is $\boxed{333 - 34 + 1 = 300}$.

Similarly, the number of numbers divisible by 5 is $\boxed{199 - 20 + 1 = 180}$.

For the final part we use a *multiple subtraction*. The number of numbers divisible by both 3 and 5 in the range is the number of numbers divisible by 15 between 105 and 990 which is (dividing by 15) $66-7+1 = 60$. Hence the number of numbers neither divisible by 3 nor 5 is

$$\boxed{900 - 300 - 180 + 60 = 480}.$$

3. **The most popular club in Harvard is, of course, the Diet Coke and Mentos Pyrotechics Club. Membership, as you know, is very exclusive. The Club always consists of 100 members: 10 drawn from the freshman, 20 from the sophomores, 30 from the juniors, and 40 from the seniors. Graduate students are vigorously barred.**

The club wishes to elect officers: the President, Vice-President, Mentos Master, and Diet Coke Panjandrum. How many ways are there to do this if no-one is allowed to hold more than one position?

How many ways are there if the Presidential position has to be filled by a senior?

How many ways are there if at least one of the four positions has to be filled by a senior?

For the first part we use the multiplication principle without repetition: $\boxed{100 \cdot 99 \cdot 98 \cdot 97}$.

For the second part we use the multiplication principle again. We imagine that the presidential position is the first to be filled, then leaving a pool of 99 for vice-president and so on.

$$\boxed{40 \cdot 99 \cdot 98 \cdot 97}$$

We use the subtraction principle for the final part. There are $60 \cdot 59 \cdot 58 \cdot 57$ ways of filling the positions using *no* seniors, hence there are $\boxed{100 \cdot 99 \cdot 98 \cdot 97 - 60 \cdot 59 \cdot 58 \cdot 57}$ ways of filling the positions using *at least one* senior.