# Homework 4 Solutions

## Problems

1. **Filling in the favorite books section of her facebook.com entry, a student chooses 4 books out of the 10 standard books of choice: Catcher in the Rye, Portnoy's Complaint, Lord of the Rings, The Great Gatsby, 1984, To Kill a Mockingbird, Catch 22, Harry Potter, Nullstellensatz, and The Sun Also Rises. How many ways are there for her to make the choice?**

   **Suppose she has actually read 1984 so she decides to definitely choose that one. How many ways are there for her to make a choice of 4 given that one of them must be 1984?**

   For the first part this is just choosing 4 unordered things from a pool of 10, hence there are $\boxed{\begin{pmatrix} 10 \\ 4 \end{pmatrix} = \frac{10!}{6!4!}}$ ways to make the choice.

   For the second part, the student has to choose a further 3 books to complete her facebook.com entry, and she now has a selection of 9 from which to choose. So there are $\boxed{\begin{pmatrix} 9 \\ 3 \end{pmatrix} = \frac{9!}{6!3!}}$ ways to make the choice.

2. **It's time to look ahead to the next season for the Harvard women's ice hockey team. Keeping Cahow, Sifers, Vaillancourt, and Wilson on the side will mean it's going to be a good one. Suppose they play 30 games in the season and end up winning 23 of those games, and losing 7. How many possible win/loss records are there? (By win/loss record I mean a list of which games were won and which were lost.)**

   **What if we drop the condition that the team win 23 games and only require that they win more than half of their games - how many possible win/loss records are there (assuming that each game ends in a win or a loss)?**

   The first part is asking how many ways can we choose 23 games from a list of 30. There are $\boxed{\begin{pmatrix} 30 \\ 23 \end{pmatrix}}$ ways of doing this.

   There is a "brute force" method for the second part, but an easier way to do it is to observe that the number of ways of winning more than half than your games is the same as the number of ways of losing more than half your games. So if we call the number of ways of winning more than half your games $N$ we have that

   $$2N + (\text{number of ways of winning exactly half the games}) = (\text{total number of possible win/loss records})$$

   Now the number of ways of winning exactly half the games is $\begin{pmatrix} 30 \\ 15 \end{pmatrix}$ and the total number of win/loss records is $2^{30}$ (can you see why?) So we see that

   $$\boxed{N = \frac{1}{2}(2^{30} - \begin{pmatrix} 30 \\ 15 \end{pmatrix})}$$