Homework 5 Solutions

Problems

1. **How many distinct ways are there to rearrange the letters in "MISSISSIPPI"? How about "WOOLLOOMOOLOO"**

   In MISSISSIPPI there are 1 M, 2 P's, 4 I's, and 4 S's. To rearrange these letters involves having 11 slots and choosing one of the slots for the M, 2 for the P's, 4 for the I's, and 4 for the S's. There are $\boxed{\binom{11}{4,4,2,1}}$ ways of doing this. Similarly for WOOLLOOMOOLOO we see there are $\boxed{\binom{13}{8,3,1,1}}$ ways of rearranging the letters.

2. **You roll two standard 6-sided dice. What is the probability that their sum is at least 8? What is the probability that their difference is at least 2?**

   There are 6^2 total possible throws. The throws whose combined score is above 8 are

   $$(2,6),(3,5),(3,6),(4,4),(4,5),(4,6),(5,3),(5,4),(5,5),(5,6),(6,2),(6,3),(6,4),(6,5),(6,6)$$

   which is 15 possible throws. Hence the probability of throwing a sum of at least 8 is $\boxed{\frac{15}{36}}$.

3. (a) To pass the time you of course play card games. What is the probability that when four cards are dealt at random from an ordinary deck of 52, each suit is present (note that a standard deck has four suits of thirteen cards each)?

   (b) Later you begin to tire of card games and together construct a rudimentary house out of palm trees and grass. In the house there is one room that would sleep 5 people, one that sleeps 4, two doubles, and a single. In this house will live your 14 "children" which are really just coconuts that you have decorated. In how many ways can you assign your children to the rooms?

   There are $52 \cdot 51 \cdot 50 \cdot 49$ possible ordered hands of 4 cards that could be dealt to you, (if we keep track of the order in which they come). A hand made up of 4 different suits can be made by first dealing any of the 52 cards to you. But then for the next card you aren't allowed to receive a card of the same suit so you have only $52 - 13 = 39$ possibilities for the second card. The third card can't be of the same suit as either of the first two cards so there are 26 possibilities for the third card. And similarly there are 13 possibilities for the final card. Hence the probability of being dealt a four-card hand with a representative of each suit is $\boxed{\frac{52 \cdot 39 \cdot 26 \cdot 13}{52 \cdot 51 \cdot 50 \cdot 49}}$

   The second part is asking how ways are there of dividing up a group of 14 children into 5 groups of 5,4,2,2, and 1. There are $\boxed{\binom{14}{5,4,2,2,1}}$ ways of doing this.