

Homework 6 Solutions

Problems

1. In the game of Yahtzee, the players roll 5 dice at a time, and the best possible roll is a “yahtzee,” which is when all 5 dice come up the same. What is the probability of rolling a yahtzee in one roll? Another valuable (and more likely) roll is the “large straight,” where the dice are all in a row (either 1,2,3,4,5 or 2,3,4,5,6). What is the probability of rolling a large straight in one roll?

There are 6^5 possible throws (keeping track of the order of the dice). Of these, there are 6 throws where all five dice come up the same. Hence the probability of throwing a yahtzee is

$$\boxed{\frac{6}{6^5}}.$$

Since the dice can come in any order, there are $5!$ ways of throwing 1, 2, 3, 4, 5, and $5!$ ways of throwing 2, 3, 4, 5, 6. Hence the probability of throwing a large straight is $\boxed{\frac{2 \cdot 5!}{6^5}}$.

2. You and a friend play the following game. You each flip three coins, and whoever gets more heads wins; if you get the same number, you win. If you win, your friend pays you \$1; if your friend wins, you pay her \$2. Who has the better odds?

There are $2^3 = 8$ possible coin sequences for each player.

If your friend gets HHH , you win 1 out of 8 times.

If your friend gets HHT , HTH , or THH , you win 4 out of 8 times.

If your friend gets HTT , THT , or TTH , you win 7 out of 8 times.

If your friend gets TTT , you win 8 out of 8 times.

In total, you win $1 + 3 \cdot 4 + 3 \cdot 7 + 8$ out of 64, i.e., your probability of winning is $\boxed{\frac{42}{64}}$. So

your friend wins with probability $\boxed{\frac{22}{64}}$, more than half of your odds. The odds of winning are in your favor, but given the payments it is a better bet for your friend.

3. In the game of Phigh, each player rolls three dice; his or her score is the highest number that appears. What is the probability of scoring 1? What is the probability of scoring 2? Your opponent scored 4. What is the probability that you'll win (that is, score 5 or 6)?

There is 1 way to score 1, out of a total of 6^3 possible rolls, so the probability is $\boxed{\frac{1}{6^3}}$.

You score 2 if you roll three 2's, two 2's and a 1, or two 1's and a 2. This can be done in $1 + 3 + 3$ ways, so the probability is $\boxed{\frac{7}{6^3}}$.

You win if you don't roll 4 or less on every die. So the probability is $\boxed{1 - \frac{4^3}{6^3}}$.