Homework 7 Solutions

Problems

1. **Flipping through your Rolodex you choose at random three friends, call them up, and find out when they were born. What is the probability that at least two of the people you chose were born on the same day of the week?**

   There are 7^3 different ways the 3 people could be distributed amongst the day s of the week. The number of ways that the 3 could be arranged so that none of them share a birthday is $7 \times 6 \times 5$, so there are $7^3 - 7 \times 6 \times 5$ ways that at least two of them share a birthday. Hence the probability is:

   $$\boxed{\frac{7^3 - 7 \times 6 \times 5}{7^3}}$$

2. **How many distinct rearrangements of "MISSISSIPPI" are possible if we don't allow the I's to be all together?**

   There are $\binom{11}{4,4,2,1}$ ways of arranging the letters in MISSISSIPPI. Let's count the number of rearrangements where the I's *are* all together and then use the subtraction principle. The clump of 4 I's could be in any of 8 positions. Once you've fixed its position there are $\binom{7}{4,2,1}$ ways of inserting the remaining letters. So, by the subtraction principle, there are

   $$\boxed{\binom{11}{4,4,2,1} - 8 \cdot \binom{7}{4,2,1}}$$

   ways of arranging the letters where the I's aren't all together. We could also just consider the clump like any other letter, in which case we might write the second term as $\binom{8}{4,2,1,1}$.