

Homework 8 Solutions

Problems

- (a) Express $\binom{7}{4} + \binom{7}{5}$ as a single binomial coefficient.

(b) Express $\binom{9}{4} + \binom{9}{3} + \binom{10}{3}$ as a single binomial coefficient.

Thinking about how the binomial coefficients fit into Pascal's triangle, and knowing that each row of Pascal's triangle is obtained from the row immediately before it by adding pairs of elements together we see that the answers are $\boxed{\binom{8}{5}}$ and

$$\binom{9}{4} + \binom{9}{3} + \binom{10}{3} = \binom{10}{4} + \binom{10}{3} = \boxed{\binom{11}{4}}$$

2. **There is life on Mars! Unfortunately, a flood is on its way. Noahzoid is eager to load up animals onto his boat. There are sheepoids, chickenoids, and zebrazoids. Each of them can reproduce asexually, but they will be even more fruitful when mating in pairs, triples, quadruples, &c., so it makes sense to load any number of them onto the boat (and maybe some won't be saved at all). If there is room for 10 animals on the boat (and there are at least 10 of each animal from which to choose), how many different combinations of sheepoids, chickenoids, and zebrazoids could be chosen by Noahzoid?**

The problem intended for exactly 10 animals to be taken aboard (rather than at most 10 animals – sorry for the ambiguity). Consider two empty trays inserted between the three groups. Their positions can be chosen in $\boxed{\binom{10+2}{2}}$ ways.