

MATH S-15, SUMMER 2001  
 GROUPS, GRAPHS, AND ALGEBRAIC STRUCTURES FOR  
 COMPUTING  
 Homework Assignment # 1  
 Due: June 27, 2001

Reading

- Section 3.6 (pp. 129–132), especially 3.51–3.53, 3.55, 3.57, and 3.61–3.62.
- Section 4.1, Definition 4.7.
- Section 4.2 (pp. 141–146), especially Examples 4.16 and 4.18.
- Section 9.4 (pp. 380–383), especially 9.40–9.58.
- Section 9.5 (pp. 389–390), Definition 9.67, Example 9.86.

Required Problems

1. Find the area of the regular  $n$ -gon with side length 1. (*Hint: Use trigonometry.*)
2. Determine the number of symmetries of a regular  $n$ -gon, and specify how many are rotations about an axis perpendicular to the plane containing the  $n$ -gon.
3. Make a table listing the symmetries for each of the five regular polyhedra (tetrahedron, cube, octahedron, dodecahedron, icosahedron), and classify them by type. (*Some of these will have been discussed in class.*)
4. Complete the table by determining the number of faces, edges, and vertices of the Platonic solids.

<i>polyhedron</i>	<i>faces</i>	<i>edges</i>	<i>vertices</i>	$F - E + V$
<i>tetrahedron</i>				
<i>cube</i>				
<i>octahedron</i>				
<i>dodecahedron</i>				
<i>icosahedron</i>				

(*Except for the last column, much of this will have been discussed in class.*)

5. Find the coordinates in  $\mathbb{R}^2$  for reasonable orientations of the following regular polygons: hexagon, octagon, dodecagon, 17-gon.
6. Let  $\sigma = (123)$  and  $\tau = (14)(25)$  be two permutations from  $S_5$ .
  - (a) Find  $\sigma \circ \tau$  and  $\tau \circ \sigma$ .
  - (b) For  $\sigma$ ,  $\tau$ , and the two permutations you computed in part (a), compute all powers  $(\sigma, \sigma^2, \sigma^3, \dots)$  until you reach the identity permutation.
  - (c) Find the inverses for  $\sigma$ ,  $\tau$ , and the two permutations you found in part (a).
7. Do exercise #1 from p. 146, section 4.2.
8. One planar representation of the regular tetrahedron is the figure below, with its vertices labeled A–D and its edges 1–6:

- (a) Write the two permutations from  $S_6$  that represent non-trivial rotations around A.
- (b) Repeat part (a) for vertices B, C, and D.
- (c) Express the other four symmetries of the regular tetrahedron as permutations from  $S_6$ . (*Hint: You may find it easiest to recognize these as products of rotations already discussed.*)

### Exploratory Problems

9. Find the coordinates in  $\mathbb{R}^3$  for reasonable orientations of the regular tetrahedron and the regular octahedron. (The dodecahedron and icosahedron are more difficult.)
10. Discuss the distinction between the symmetry groups of planar objects and those of solid objects. In particular, why are reflections considered valid symmetries of planar objects, but not of solid objects? (What *is* a reflection of a solid object?) How would the symmetry groups be different if we did/did not allow reflections?
11. Build physical models of the five regular polyhedra.