MATH S-15, SUMMER 2001
GROUPS, GRAPHS, AND ALGEBRAIC STRUCTURES FOR
COMPUTING
Homework Assignment # 10
Due: August 1, 2001

Reading

Sections 6.1-6.3 and 6.5-6.6.
In preparation for next time, Sections 14.2-14.4.

Required Problems

1. Consider the graphs in problem # 2 from p. 251 of Section 6.5.

   (a) Which of the four have Euler paths? For each graph that has an
       Euler path, give one example.
   (b) Which of the graphs are connected? Explain.

2. Problem # 4 from p. 251 of Section 6.5.

3. Problem # 7 from p. 251 of Section 6.5.

4. Problems # 2 and 4 from p. 261 of Section 6.6.

5. Problem # 8 from p. 261 of Section 6.6.

6. Problem # 10 from p. 261 of Section 6.6.

7. Problem # 2ace from p. 538 of Section 14.1.

8. How many edges are in the graph K_n, the complete graph on n vertices?
   How many edges are in the graph $K_{m,n}$, the complete bipartite graph
   with sets of m and n vertices?

9. Show that if G is a tree with e edges and v vertices, then $v = e + 1$.

Exploratory Problems

10. Prove that in any simple graph with two or more vertices, there must be two vertices with the same degree.

11. Can a citizen of Konigsberg make a tour of the city crossing each of the bridges exactly twice? Explain. Can the mayor help solve the original problem by building another bridge? Explain.

12. Problem # 4abcd from p. 540 of Section 14.1.

13. Show that a connected graph is bipartite if and only if it does not have a cycle of odd length.

14. Let G be a graph with $n \geq 2$ vertices.

    (a) Prove that if G has at least $\binom{n-1}{2} + 1$ edges, then G is connected.

    (b) For any $n \geq 2$, find a graph G with $\binom{n-1}{2}$ edges that is not connected.