MATH E-105, SUMMER 2001
GROUPS, GRAPHS, AND ALGEBRAIC STRUCTURES FOR COMPUTING
Homework Assignment # 11
Due: August 6, 2001

Reading

For this week's material, Sections 14.1–14.5.
For next time, Sections 15.1 and 15.5.

Required Problems

1. Problems # 15 and 16 from p. 549 of Section 14.1.
2. Problem # 1bcd from p. 556 of Section 14.2.
3. Problem # 6 from p. 557 of Section 14.2.
4. Show that if G is a planar graph that does not contain $K_{3,3}$ as a subgraph, then $e \leq 2v - 4$, where e is the number of edges of G and v is the number of vertices of G.
5. Problem # 4abcd from p. 567 of Section 14.3.
6. Determine the chromatic numbers of the graphs of the five Platonic solids, where vertices and edges of the polyhedra correspond to vertices and edges of the graphs, respectively.
7. Problem # 2abd from p. 576 of Section 14.4. (If the graph has no Hamiltonian cycle, explain why not.)
8. Problem # 4acde from p. 577 of Section 14.4. (If the graph has no Hamiltonian path, explain why not.)
9. Give an example of a graph with six vertices that has an Euler cycle but no Hamiltonian cycle and an example of a graph with six vertices that has a Hamiltonian cycle but no Euler cycle.
10. Problem # 1abc from p. 588 of Section 14.5.

Exploratory Problems

11. Problem # 24 from p. 549 of Section 14.1.

12. Problem # 26 from p. 549 of Section 14.1.

13. Problem # 8 from p. 557 of Section 14.2.

14. Problem # 10 from p. 557 of Section 14.2.

15. Problem # 10 from p. 568 of Section 14.3.

16. A soccer ball is made by stitching together pieces of leather which are either regular pentagons or regular hexagons and which all have the same side lengths. Show that a properly made soccer ball must have 12 pentagons.

17. Draw *planar* graphs corresponding to the five Platonic solids, and explain why Euler's formula applies. Generalize a method for drawing a planar graph corresponding to any polyhedron inscribed in a sphere. Does the same result hold for a polyhedron inscribed in an ellipsoid? Inside any convex surface? Does the same result hold for polyhedra inscribed in a single-holed torus (i.e. a donut)? How general is Euler's formula?