MATH S-15, SUMMER 2001
GROUPS, GRAPHS, AND ALGEBRAIC STRUCTURES FOR
COMPUTING
Homework Assignment # 12
Due: August 13, 2001

Reading

Sections 14.4, 14.5, 15.1, 15.5, and 15.6.

Required Problems

1. Problem # 2abcd from p. 243 of Section 6.3.

2. Show that every tree with at least $n \geq 2$ vertices is bipartite.

3. Problem # 2abd from p. 576 of Section 14.4. (If the graph has no Hamiltonian cycle, explain why not.)

4. Problem # 4acde from p. 577 of Section 14.4. (If the graph has no Hamiltonian path, explain why not.)

5. Give an example of a graph with six vertices that has an Euler cycle but no Hamiltonian cycle and an example of a graph with six vertices that has a Hamiltonian cycle but no Euler cycle.

6. Problem # 1abc from p. 588 of Section 14.5.

7. In problem # 4 from p. 595 of Section 15.1, draw the trees when vertices b and j are selected as roots (so, there should be two different trees!), and answer parts (i)-(vi) for each of these trees.

8. Problem # 8 from p. 596 of Section 15.1.

9. How many spanning trees are there for the (labelled) graph K_7, the complete graph on 7 vertices?

10. How many spanning trees are there for the (labelled) graph $K_{2,n}$, the complete bipartite graph with sets of 2 and n vertices?

11. Problem # 4 from p. 637 of Section 15.5.

12. Problem # 10 from p. 639 of Section 15.5.

13. Problem # 2ab from p. 648 of Section 15.6.

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For Problems 14–17, consider the weighted graph titled `united.aaa`:

14. Suppose you purchase a United Airlines "Graphic Flyer" pass that allows you to fly anywhere you wish along the routes shown on the diagram.

    (a) List the order in which you will visit the cities if you start from Boston and do a preorder depth-first traversal.

    (b) List the order in which you will visit the cities if you start from Boston and do a postorder depth-first traversal.

    (c) List the order in which you will visit the cities if you start from Boston and do a breadth-first traversal.

15. A cheaper version of the same pass, the "Millenium Freedom Fare" requires that you restrict yourself to flights that are part of a minimal spanning tree for the graph. List the order in which flights will be added to this tree if you build it by using:

    (a) Prim's algorithm, starting from Boston.

    (b) Kruskal's algorithm.

16. Suppose you set out to determine the shortest distance from Boston to every other city in the United Airlines graph, using Dijkstra's algorithm. Make a table showing the order in which the cities are marked "visited" and the minimum distance to each.

17. Using the results of Problem # 14, show how you would discover the articulation points of United Airlines starting from Boston. Draw a diagram like the one in the notes that shows the depth-first spanning tree that starts from Denver.

18. Starting in one corner and making "legal" moves (i. e. the possible moves provided by the rules of chess, despite the absence of other pieces from the board), is it possible for a knight to tour the standard 8×8 chessboard? 7×7? 6×6?

19. In a group of $2n$ people, each of whom knows at least n others, prove that it is possible to seat the people at a round table such that each person next to at least one person known to him or her.

20. Problems # 14 and 15 from p. 596 of Section 15.1: For each of the two following statements, provide either a proof or a counter-example:

    Every edge of a tree is a cut edge.

    Every vertex of a tree is a cut vertex.

21. Problem # 18 from p. 640 of Section 15.5.

22. How many non-isomorphic rooted trees with n vertices are there? Explain. (For a partial answer, read Theorem 15.12 carefully.)

23. If we replace distances by tolls for a highway graph, it becomes possible to imagine negative-cost edges: toll stations at which the motorist receives money from the attendant! We can consider the problem of finding the minimum-toll path between two cities.

    (a) Explain why this problem has no meaningful solution if a graph contains a cycle of negative net cost.

    (b) If there is at least one road with negative toll but no cycle of negative net toll, then Dijkstra's algorithm may fail to find the lowest-cost path. Construct a graph that illustrates this claim.

24. Consider the problem of constructing a maximum-cost spanning tree. Invent a variant of either Kruskal's algorithm or Prim's algorithm to solve this problem.