Solutions to Math Xa 2^{nd} Exam: Dec 11, 1995

1

a) $(\ln x)^2 = 1$

$\ln x = \pm 1$

$x = e^{\pm 1}$

$\boxed{x = e, 1/e}$

b) $\log x^4 + \log\sqrt{x} = 3$

$4\log x + \frac{1}{2}\log x = 3$

$\frac{9}{2}\log x = 3$

$\log x = \frac{2}{3}$

$\boxed{x = 10^{2/3}}$

c) $(e^x)^3 = e^{x^3}$

$e^{3x} = e^{x^3}$

$3x = x^3$ Common Error - If you cancel the x's you'll lose a root.

$x^3 - 3x = 0$

$x(x^2 - 3) = 0$

$\boxed{x = 0, x = \pm\sqrt{3}}$

d) $5(3^{2x+1}) = 2(4^x)$

$5 \cdot 3 \cdot (3^2)^x = 2 \cdot 4^x$

$\frac{15}{2} = \frac{4^x}{9^x} = \left(\frac{4}{9}\right)^x$

$\ln\frac{15}{2} = x\ln\left(\frac{4}{9}\right)$

$\boxed{x = \frac{\ln\left(\frac{15}{2}\right)}{\ln\left(\frac{4}{9}\right)}}$

2

a) $y = 5\ln(7x^2) + 5e^{-3x} + \ln(5x^{-2})$

$y = 5\ln 7 + 10\ln x + 5e^{-3x} + \ln 5 - 2\ln x$

$y = 8\ln x + 5(e^{-3})^x + \ln 5 + 5\ln 7$

$\boxed{y' = \frac{8}{x} - 15e^{-3x}}$

b) $f(x) = 5 \cdot 3^{t/2} = 5 \cdot (\sqrt{3})^t$

$f'(x) = 5\ln(3^{1/2})(3^{t/2})$

$\boxed{f'(x) = \left(\frac{5}{2}\ln 3\right)(3^{t/2})}$

$f''(x) = \left(\frac{5}{2}\ln 3\right)\left(\frac{1}{2}\ln 3\right)3^{t/2}$

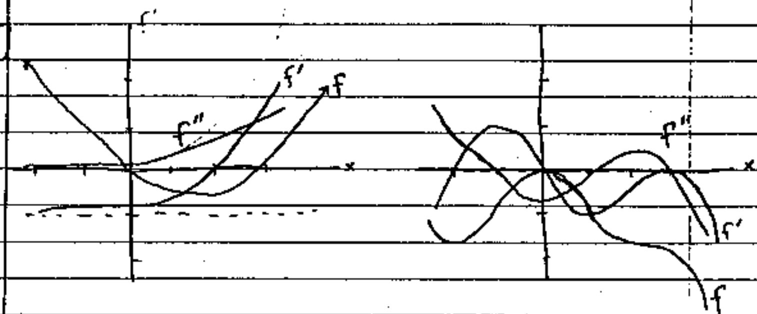
$\boxed{f''(1) = \frac{5}{4}(\ln 3)^2\sqrt{3}}$

3

a) $y = g\frac{(x-f)(x-a)(x-c)}{x(x-b)^2}$

b) $y = \frac{c(x-a)^2(x-b)}{-a^2 b}$

4



5 $f'(5) = 0$

a) $f''(x) = e^x$

$f'(5) = e^5 > 0 \Rightarrow$ concave up

$x = 5$ local min

b) $f(x) = x^2 - 10x + 25$

$f'(x) = 2x - 10 \rightarrow f''(x) = 2 > 0$, so concave up

$x = 5$ local min

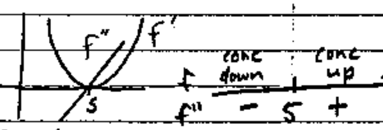
c) $f''(x) = (x-5)^3$

f: conc down | conc up

f'': $-$ 5 $+$

inflection point

d)



inflection point

e) $f''(5) = 0$

$f'(5) = 0$

can't tell

6 i) A: $(\ln 5, 5)$
B: $(1, e)$
C: $(e, 1)$
D: $(e^2, 2)$

ii) a) $g'(x) = e^x$
$g'(1) = e$
b) $g''(x) = e^x$
$g''(\ln 5) = 5$
c) $f'(x) = 1/x$
$f'(e) = 1/e$

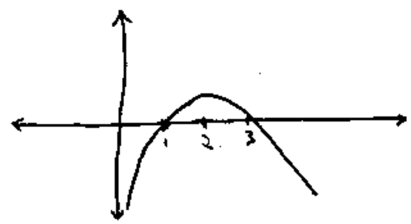
7 $P = P_0 e^{kt}$
a) $P_0 = 20$
$20e^{3k} = 35$
$e^{3k} = \frac{35}{20}$
$k = \frac{1}{3}\ln\left(\frac{35}{20}\right) \approx 0.1864$
$\boxed{P(t) \approx 20e^{.1864t}}$

b) $20e^{.1864t} = 50$
$e^{.1864t} = 5/2$
$\boxed{t = 4.916 \text{ years}}$
c) $P'(0) = .1864 \cdot 20 = \boxed{3.728 \text{ bears/year}}$
$P'(3) = \boxed{6.521 \text{ bears/year}}$

8 $f(x) = 4\ln x - (x-1)^2 = 4\ln x - (x^2 - 2x + 1)$
a) Domain is $x > 0$ (or $\ln x$ is undefined)
b) $f'(x) = \frac{4}{x} - 2x + 2$, set $f'(x) = 0$ to find critical points: $\frac{4}{x} - 2x + 2 = 0 \Rightarrow 4 - 2x^2 + 2x = 0$
or $x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$\Rightarrow x = 2$ and $x = -1$
but only $x = 2$ is in the domain
at $x = 2$ $f'(2) = 0$, and check $f''(x) = -\frac{4}{x^2} - 2$
since $f''(2) = -1 - 2 < 0 \Rightarrow$ concave down at $x = 2 \Rightarrow$ local max. Note in fact $f''(x) < 0$ for all x so concave down everywhere $\Rightarrow$ $x = 2$ is a global (absolute) max

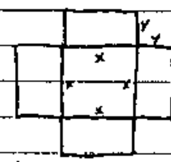
8 continued (c) $f'(x) = \frac{4}{x} - 2x + 2$ is a continuous function for $x > 0$ (in the domain of $f(x)$), and has a zero at $x = 2$
check sign of f': $\underset{0}{+} \; \underset{2}{-}$ so f increasing for $0 < x < 2$, f is decreasing for $2 < x$
(d) $f''(x) = \frac{-4}{x^2} - 2$ is always negative, so $f'(x)$ is decreasing for all x
(e) trick question! range of $f(x)$ is $(-\infty, 4\ln 2 - 1)$, so there is no absolute minimum!

(f)



9 a) $y = 3 \cdot 2 \cdot 2^{-x} = 6 \cdot \left(\frac{1}{2}\right)^x$ Interchange x+y.
$x = 6 \cdot \left(\frac{1}{2}\right)^y \Rightarrow x/6 = \left(\frac{1}{2}\right)^y \Rightarrow \ln\frac{x}{6} = y \ln\frac{1}{2}$
$\Rightarrow y = \boxed{\frac{\ln x/6}{\ln 1/2}} = g^{-1}(x)$
b) $D[g^{-1}(x)] = R[g(x)] = (0, \infty)$
$R[g^{-1}(x)] = D[g(x)] = (-\infty, \infty)$

10



$V = x^2y = 10$
$y = \frac{10}{x^2}$
Cost $= .15x^2 + 6(4xy)$
$C(x) = 15x^2 + 24x\left(\frac{10}{x^2}\right)$
$C(x) = 15x^2 + \frac{240}{x}$
$C'(x) = 30x - \frac{240}{x^2} = 0$ $\boxed{x = 2}$ $\boxed{y = \frac{10}{x^2} = \frac{10}{4}}$
$x^3 = \frac{240}{30} = 8$ $C = 15(4) + \frac{240}{2} = \boxed{\$180}$