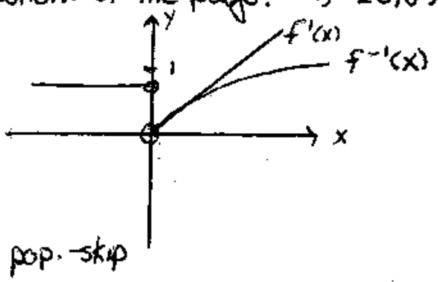
Errors in the Review Packet

- **On Solutions to Problems from Old First Semester Exams**

p.1 #1 at the bottom of the page: a) $[0,5)$ not $[3,5)$

p9 #6

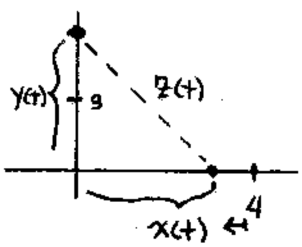


p7 #7 Mexico City pop. - skip

- **On Solutions to Xa Final 1/8/95** #10 last line: In the 12th month: December 1996

Also, the answer to #4 is right but uses a method we haven't used yet.
Here it is using methods we know:

#4.



$x(t) = 4 - 8t$ rate × time = distance

$y(t) = 3 + 9t$

so $z(t) = \sqrt{[x(t)]^2 + (y(t))^2}$

we want $z'(t)$ when $t=0$ (At $t=0$ $x(t)=4$ & $y(t)=3$)

$$z(t) = \sqrt{(4-8t)^2 + (3+9t)^2}$$

$$z'(t) = \frac{1}{2}\,\frac{1}{\sqrt{(4-8t)^2+(3+9t)^2}}\cdot\left[(2(4-8t)(-8) + 2(3+9t)\cdot 9\right]$$

$$z'(0) = \frac{1}{2}\,\frac{1}{\sqrt{4^2+3^2}}\cdot\left[2\cdot 4\cdot(-8) + 2\cdot 3\cdot 9\right]$$

$$z'(0) = \frac{1}{2}\,\frac{1}{5}\left[-64 + 54\right] = \frac{-10}{10} = -1$$

The distance is decreasing at a rate of 1 mph.

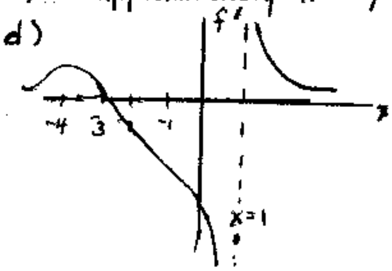
Answers to Jan. 15, 1998 Xa Final
No Explanations -
just answers.

① a) $(-3,0)$

b) $(-4,1)$ and $(1,\infty)$

c) at approximately $x=-4$

d)



② a) $f'(x)=\pi\cdot(-\frac{1}{2})(2x^{\pi}+x)^{-3/2}\cdot(2\pi x^{\pi-1}+1)$

b) $-\frac{1}{(\ln(x^2+1))^2}\cdot\frac{1}{x^2+1}\cdot 2x$

c) $\frac{5}{4}(\ln 2\cdot 2^t\cdot t^7+7t^6\cdot 2^t)$

d) $3\left(\frac{1}{x}+\frac{1}{2x}-\frac{5}{5x+1}\right)$

③ a) $\frac{1}{10}$

b) $g'(4)=\lim_{h\to 0}\frac{\frac{4+h}{4+h+1}-\frac{4}{5}}{h}=\dots$

$\lim_{h\to 0}\frac{1}{h}\left(h\frac{(5-4)}{25+5h}\right)=\frac{1}{25}$

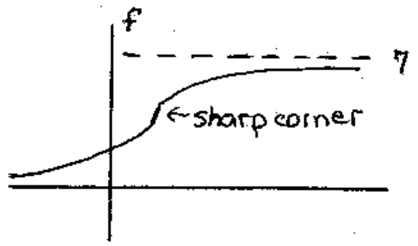
c) Use product rule $g(t)=t(t+1)^{-1}$

$g'(t)=\dots=\frac{1}{(t+1)^2}$

$g'(4)=\frac{1}{(4+1)^2}=\frac{1}{25}$

④ C, D, A, E, B

⑤



⑥ $p'(x)=3[h(g(x))+\pi]^2\cdot[h'(g(x))\cdot g'(x)]$

b) $\frac{dp}{dx}\Big|_{x=2}=99[169+26\pi+\pi^2]$

c) i) $\frac{dp}{dx}$ is always positive

⑦ Max: doctors: $11/6$

Nurses: 5.5

Show that this gives a max

⑧ a) $y=\frac{2x(x-3)^2}{(x-1)(x+2)}$

b) $y=2(\frac{1}{3})^x-18$

⑨ a) $x=0$

b) critical pts:

$x=0$ neither; $x=\frac{3}{b}$ local max

c) Global max at $x=\frac{3}{b}$

Value: $f(\frac{3}{b})=\frac{27}{b^3}e^{-3}=\frac{27}{b^3e^3}$

d) No

⑩ a) $\ln[P_0e^{kt}]$

b) i)

c) $\ln P_0$

d) Neg. if $P_0<1$

0 if $P_0=1$

Pos. if $P_0>1$

e) It's k. It's constant

f) $w=e^{10}\cdot e^{-2x}$

⑪ a) i

b) i

c) ii

d) ii

e) $h^{-1}(1)$ gives the volume of the milk when the height is 1.

Answers to Jan 25, 1999 Xa Final
No Explanations, Just Answers

① a) $f'(x) = 7(3x^3 e^{x^3} + e^{x^3})$

b) $g'(x) = \dfrac{2}{10x(\ln x^2)^{9/2}}$

c) $h'(x) = \dfrac{-3^x(4\pi x^3) + \ln 3 \cdot 3^x \cdot \pi x^4}{(\pi x^4)^2}$

② a) When the company spends half as much on advertising as it did last year, it receives \$80,000 less in revenue than it did last year.

b) When \$30,000 are spent on advertising, the rate of change of revenue received per dollar spent is 2.8.
When spending \$30,000 on ads the additional revenue expected for spending an additional dollar is approx. \$2.8.

c) The amt the company should spend on advertizing in order to double last yrs' revenue.

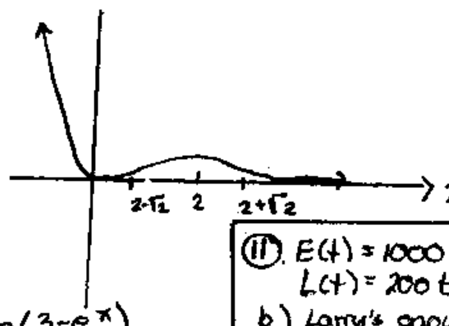
③ 2 in × 2 in × 5 in. Must show this is actually minimizing cost.

④ a) decreasing on $(-\infty, 0)$ $(2, \infty)$
increasing on $(0, 2)$

b) concave up on $(-\infty, 2-\sqrt{2})$ and $(2+\sqrt{2}, \infty)$

concave down on $2-\sqrt{2} < x < 2+\sqrt{2}$

c) Absol. min at (0,0)
Absolute max - None.



⑤ a) $y = \ln(3 - e^x)$

b) $y = e^{\left(\frac{x-2}{2}\right)}$

c) $y = e^{\left(\frac{x - \ln 3}{2}\right)}$

d) $y = \frac{1}{2}$

⑥ b, e, f, h

⑦ a) sometimes
b) always
c) sometimes
d) sometimes
e) always
f) sometimes
g) never

⑧ a) stretch vertically by a factor of 2
b) shift left 2
c) shrink horizontally by a factor of 2

sketch

$a'(3) = 3$
$b'(-2) = 0$
$c'(-2) = 2f'(-4) = 2(\frac{1}{2}) = 1$

⑨ a) $f'(x) = \dfrac{1}{(x+1)^2}$

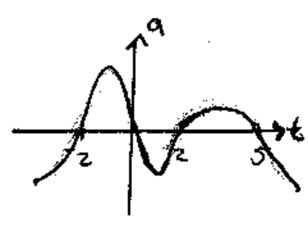
b) $f'(2) = \lim_{h \to 0} \dfrac{f(2+h) - f(2)}{h}$

$= \lim_{h \to 0} \left[\dfrac{2+h}{2+h+1} - \dfrac{2}{3}\right] \dfrac{1}{h}$

$= \lim_{h \to 0} \left[\dfrac{(2+h)3 - 2(3+h)}{(3+h) \cdot 3}\right] \dfrac{1}{h}$

$= \dots = \frac{1}{9}$

⑩ a) $t = 3$
b) $t = 3$
c) $t = -2$
d) i) concave up on $(-2, 0)$, $(2, 5)$
where $v(t)$ is increasing the graph of $s(t)$ is concave up
ii) $t = 3$
$t = 3$
$t = -2, 0, 2, 5$



⑪ $E(t) = 1000 \cdot 2^{t/5}$
$L(t) = 200t + 1000$

b) Larry's grows quicker at $t = 0$: $L'(0) = 200$

c) Earl's is growing more rapidly.
$E'(5) = 2000 \ln 2^{1/5} = 100 \ln 2$.

d) (iii) Maximize $L - E$.
$(L-E)' = 0 \Rightarrow L' = E'$