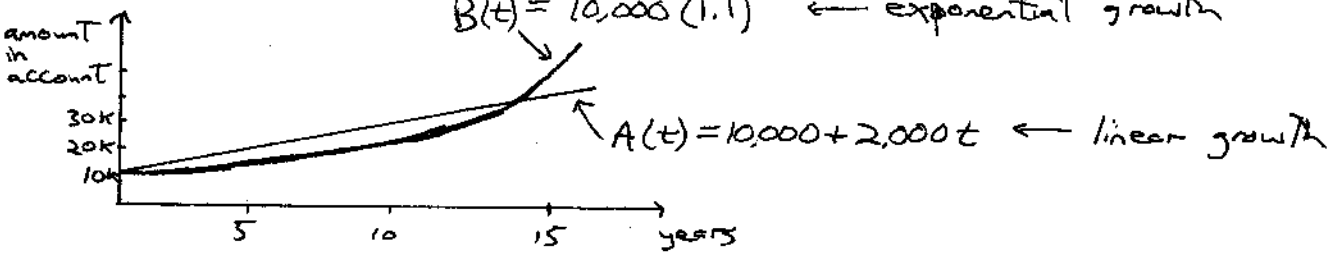
Answers to Handout from Monday's Exponential Growth and Decay Class.

(A) Gov't Soc Security Plan

(1) Add-on plan: if start with 10,000, then 20% of 10,000 = $2,000 will be added each year. Total after t years is $A(t) = 10{,}000 + 2{,}000t$

(2) If get 10% increase to amount each year, then the original amount gets multiplied by $100\% + 10\% = 110\% = 1.1$ each year, so total in account after t years is $10{,}000(1.1)^t = B(t)$

Now use a graphing calculator to compare the two functions:



Up until about 14 years, when the amount in both plans is equal (i.e. $A(14) \approx B(14)$), then the linear add-on plan is preferable. After that the exponential function grows far more rapidly, and is much more desirable.

Or you could make a table of values:

years	Add-on Plan $A(t) = 10{,}000 + 2{,}000t$	Plan #2 $B(t) = 10{,}000(1.1)^t$
0	$10,000	$10,000
1	12,000	11,000
2	14,000	12,100
⋮		
5	20,000 ← larger amounts with add-on plan	16,105.10
10	30,000	25,937.42
⋮		
14	38,000	37,975
40	$90,000	$452,592 ← Plan #2 a winner by far!

(B) Ozone Problems

Using the data table shown you can quickly rule out a linear function model, as the differences aren't the same (linear → change is constant per unit (year) change) For instance, from 1988 to 1990 (a difference of 2 years) the ozone decreased from 64 to 57.76 ppm, a decrease of 6.24, whereas from 1990 to 1992, also a difference of two years, the ozone dropped from 57.76 to 52.13 ppm, a decrease of 5.63 ppm (which is not equal to 6.24).
Thus, try an exponential function model. We'd like to find the ratio of two consecutive years to find the multiplier involved, so try 1992 → 1993, decrease from 52.13 to 49.52, $\frac{49.52}{52.13} \approx .95$

Now to check that this works for our data we can start with 64, the amount in 1988, then multiply by .95 for each year that passes and check to see if this gives the same results as in the table (check it → it does).
So a formula in this instance would be $64 \cdot (.95)^t$ where t measures years passed since 1988.

in year 2000, $t = 12$ (years since 1988), and the ozone would have decreased to $64 \cdot (.95)^{12} \approx 34.58$ ppm

Now either graph $64(.95)^t$ and find when it falls to below 25, or continue to make a table of values as t gets larger:

in 2000	$t = 12$	$64(.95)^{12} \approx 34.58$
2001	13	32.85
2002	14	31.21
2003	15	29.65
2004	16	28.17
2005	17	26.75
2006	18	25.42